How robust is the long-run relationship between temperature and radiative forcing?

Terence C. Mills

Abstract This paper examines the robustness of the long-run, cointegrating, relationship between global temperatures and radiative forcing. It is found that the temperature sensitivity to a doubling of radiative forcing is of the order of $2 \pm 1°C$. This result is robust across the sample period of 1850 to 2000, thus providing further confirmation of the quantitative impact of radiative forcing and, in particular, CO_2 forcing, on temperatures.

1 Introduction

With the publication of both Sir Nicholas Stern's Review on the Economics of Climate Change (Stern 2007) and the IPCC Fourth Assessment Report (IPCC 2007), attention has again focused on the impact of radiative forcings, notably those of carbon dioxide, on temperature. In a recent paper, Kaufmann et al. (2006a) examine the cointegrating relationship between global temperature and the radiative forcing of greenhouse gases and anthropogenic sulphate emissions. Although their analysis also focuses on feedbacks from temperature to emissions and on short-run dynamics, a key finding is their estimate of the temperature sensitivity implied by the cointegrating regression. This is defined as the temperature response to a doubling of radiative forcing (interpreted by them as a doubling of CO_2 concentration) and is estimated to be of the order of 1.7–3.5°C, the exact value depending on the sample period actually employed. This temperature sensitivity is regarded as being consistent with the transient temperature response from a simulated climate model, defined as the temperature change observed at the point when the atmospheric concentration of carbon dioxide has doubled in a climate model simulation in

T. C. Mills (✉)
Department of Economics, Loughborough University,
Loughborough, Leics, LE11 3TU, UK
e-mail: t.c.mills@lboro.ac.uk

which concentrations increase by 1% per annum compounded over seventy years ($1.01^{70} \approx 2$; see Kaufmann et al. 2006b).

The purpose of the present paper is to examine the robustness of this important result by using an extended and updated data set and more detailed sub-period and structural break analysis. To this end, the data set is discussed in Section 2, empirical analysis is reported in Sections 3 and 4, and some conclusions are drawn in Section 5.

2 Data

The global temperature data are from Brohan et al. (2006) and are near-surface observations taken on land and sea. The radiative forcings are taken from Stern (2005) and cover the period from 1850 to 2000, somewhat longer than the 1860–1994 period studied by Kaufmann et al. (2006a). The global temperature series, denoted henceforth as g_t, is shown as Fig. 1 and reveals the familiar pattern of a rising, but by no means monotonic, trend throughout the twentieth century. Stern (2005) provides data, measured in watts per metre^2 (w/m^2), on seven radiative forcings: carbon dioxide ($co2_t$), methane ($ch4_t$), nitrous oxide ($n2o_t$), CFC11 and CFC12 ($cfc11_t$ and $cfc12_t$; both zero until 1938 and 1949 respectively), anthropogenic sulphur aerosols (sox_t), and solar irradiance ($solar_t$). These are shown in Fig. 2 and display a variety of magnitudes and evolutions. For example, carbon dioxide, methane and nitrous oxide forcings have increased secularly throughout the period whereas sulphur aerosol forcing declined until the late 1980s, since when it has increased somewhat. Solar irradiance forcing shows a slight secular increase across the period, although the dominant feature is its familiar oscillation with a period of approximately eleven years. Similar to Stern and Kaufmann (2000), we consider four aggregates of radiative forcing calculated from these components:

(a) Total

$$rf\_tot_t = co2_t + ch4_t + n2o_t + cfc11_t + cfc12_t + sox_t + solar_t$$

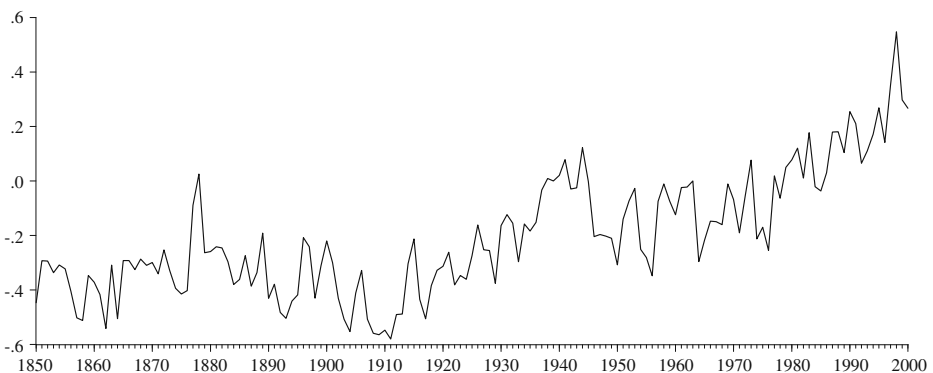(b) Anthropogenic

$$rf\_anth_t = rf\_tot_t - solar_t$$

Fig. 1 Global temperature, 1850–2000, measured as anomalies from 1961–1990 mean
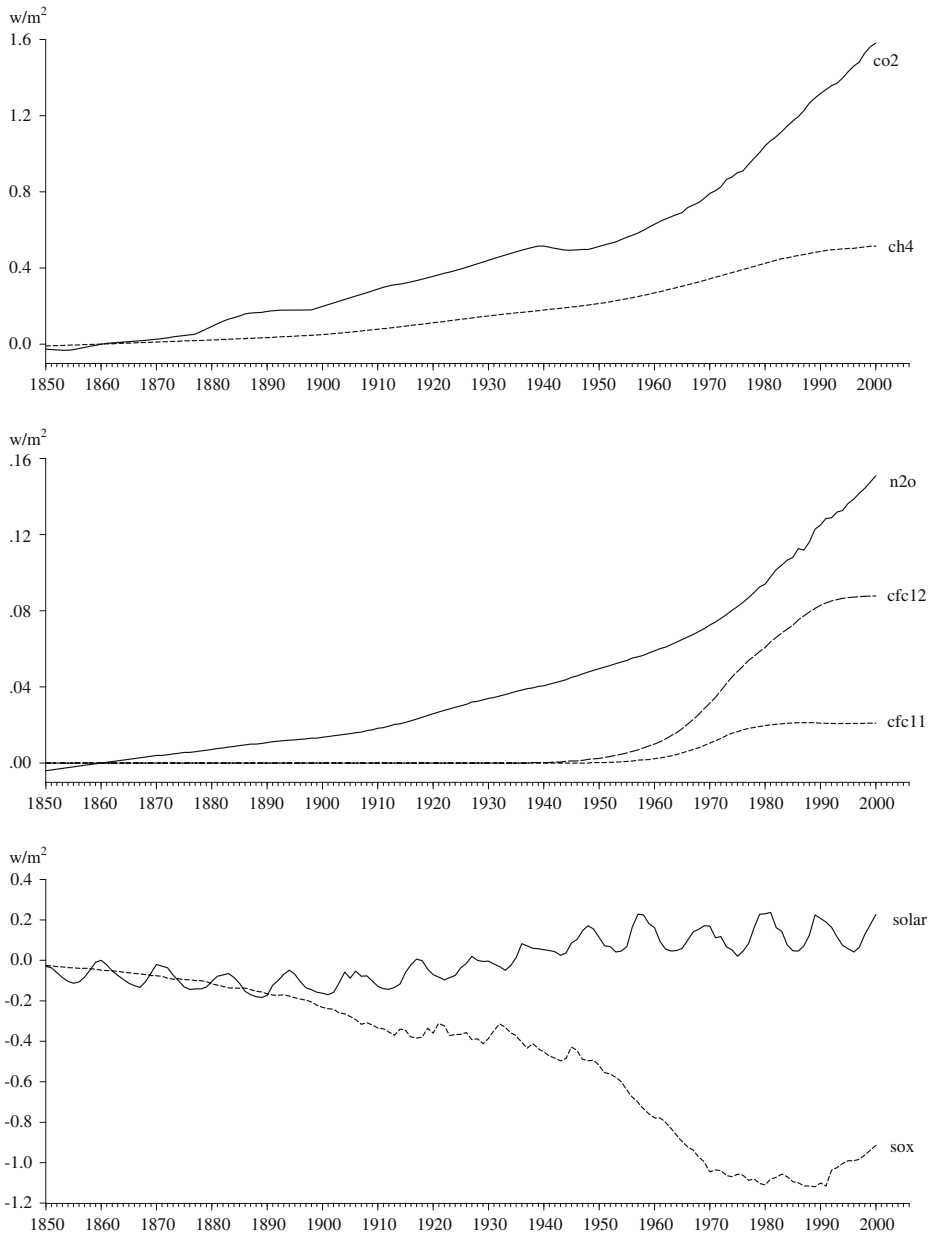
Fig. 2 Component radiative forcings, 1850–2000

(c)   Greenhouse gas

$$rf\_ghg_t = rf\_anth_t - sox_t$$

(d)   Greenhouse gas excluding carbon dioxide

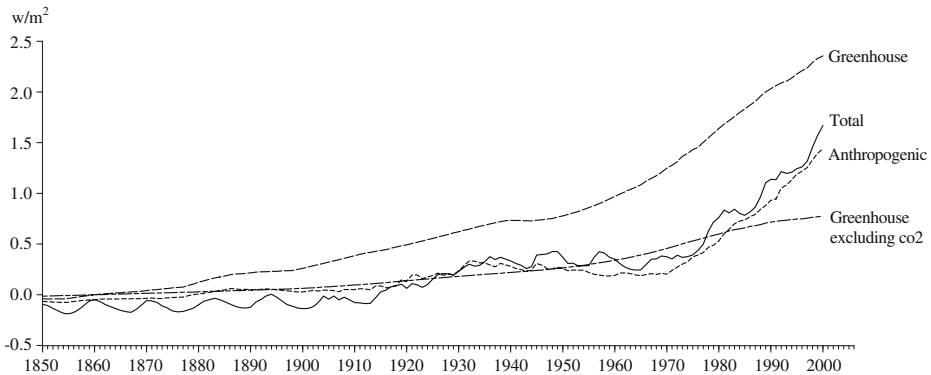$$rf\_ghg_t - co2_t = rf\_ghg_t - co2_t.$$

Fig. 3 Aggregate radiative forcings, 1850–2000

These are shown in Fig. 3 and, although all increase throughout the 20th century, the rates of increase are rather different depending on which radiative forcings are included and excluded: average annual increases over the twentieth century are 0.018 for *rf_tot*, 0.014 for *rf_anth*, 0.021 for *rf_ghg*, but only 0.007 for *rf_ghg-co2*, thus reflecting the dominant impact of carbon dioxide emissions.

3 Full sample cointegration analysis

Kaufmann and Stern (2002) and Kaufmann et al. (2006a) provide evidence of cointegration between temperature and radiative forcing. These authors consider the potential cointegrating relationship

$$g_t = \alpha_1 + \beta_1 rf\_tot_t + u_{1t} \tag{1}$$

This specification assumes that the temperature effect of a unit of radiative forcing (i.e., 1 w/m²) is equal across all forcings. While this assumption is consistent with physical theory (see Kaufmann et al. 2006a), we can test it if we also consider further potential cointegrating relationships based on the three sub-aggregates defined above:

$$g_t = \alpha_2 + \beta_2 rf\_anth_t + \gamma_2 solar_t + u_{2t} \tag{2}$$

$$g_t = \alpha_3 + \beta_3 rf\_ghg_t + \gamma_{31} solar_t + \gamma_{32} sox_t + u_{3t} \tag{3}$$

$$g_t = \alpha_4 + \beta_{41} rf\_ghg_t - co2_t + \beta_{42} co2_t + \gamma_{41} solar_t + \gamma_{42} sox_t + u_{4t} \tag{4}$$

If the physical assumption of equal impact of forcings is correct then the slope coefficients in each of these equations should be equal, and this is a hypothesis that can be tested. How it may be tested will depend on whether Eqs. 1–4 form cointegrating relationships. This in turn depends upon whether the error processes u_{it}, $i = 1, \ldots, 4$, are stationary (i.e., are I(0)). The stationarity of the error processes is tested by the Engle and Granger (1987) ECM statistic, which is obtained by

Table 1 Cointegration test statistics

	$\hat{u}_1$	$\hat{u}_2$	$\hat{u}_3$	$\hat{u}_4$
$t(\rho = 0)$	−6.84 [0]	−6.92 [0]	−6.92 [0]	−7.07 [0]
0.001 critical value	−5.19	−5.52	−5.81	−6.04

Value in [ ] denotes k, the order of lag augmentation in (5)

subjecting the OLS residual $\hat{u}_{it}$ to an augmented Dickey–Fuller (ADF) test, i.e., by fitting the following regression (where $\Delta\hat{u}_{it} = \hat{u}_{it} - \hat{u}_{i,t-1}$ is the first difference of $\hat{u}_{it}$)

$$\Delta\hat{u}_{it} = \rho\hat{u}_{i,t-1} + \sum_{j=1}^{k} \delta_{ij}\Delta\hat{u}_{i,t-j} + e_{it} \qquad (5)$$

and testing whether ρ is significantly negative with a t-statistic, using critical values calculated by MacKinnon (1996). The test statistics reported in Table 1 confirm that in each case $\hat{\rho}$ is significantly negative (at levels less than 0.001), so that the residuals are indeed stationary.

Given that each equation is thus a cointegrating relationship, efficient estimation requires determining the integration properties of the variables appearing in each equation. These are determined by ADF tests of the form 5 applied to the levels and the first differences of each variable (for the levels a time trend is included as an additional regressor). From Table 2, it is seen that, for levels, $\rho = 0$ cannot be rejected for any series, so that all are at least I(1). For the differences, $\rho = 0$ cannot be rejected for rf_ghg, $rf\_ghg\text{-}co2$ and $co2$ itself, so that these series appear to be I(2).

Since all the variables appearing in Eqs. 1 and 2 are thus I(1), these equations can therefore be estimated efficiently using either Stock and Watson's (1993) DOLS or DGLS estimator applied to

$$g_t = \alpha_1 + \beta_1 rf\_tot_t + \sum_{j=-p}^{p} \phi_j \Delta rf\_tot_{t-j} + u_{1t}$$

and

$$g_t = \alpha_2 + \beta_2 rf\_anth_t + \gamma_2 solar_t$$
$$+ \sum_{j=-p}^{p} \left(\phi_{21j}\Delta rf\_anth_{t-j} + \phi_{22j}\Delta solar_{t-j} \right) + u_{2t}$$

Table 2 ADF test statistics

	Levels	Differences
g	−2.52 [3]	−11.18 [2]
rf_tot	1.61 [4]	−6.06 [4]
rf_anth	3.81 [1]	−4.57 [2]
rf_ghg	0.28 [3]	−1.22 [3]
$rf\_ghg\text{-}co2$	−3.11 [3]	−1.18 [2]
$solar$	−2.01 [8]	−10.89 [6]
sox	−1.25 [1]	−9.46 [0]
$co2$	1.82 [2]	−1.64 [2]
10% critical value	−3.14	−2.58
5% critical value	−3.44	−2.88

Value in [ ] denotes k, the order of lag augmentation in (5)

In Eq. 3 only rf_ghg is I(2), so that the equation to be estimated by either DOLS or DGLS is

$$g_t = \alpha_3 + \beta_3 rf\_ghg_t + \gamma_{31} solar_t + \gamma_{32} sox_t$$
$$+ \sum_{j=-p}^{p} \left(\phi_{31j}\Delta^2 rf\_ghg_{t-j} + \phi_{32j}\Delta solar_{t-j} + \phi_{33j}\Delta sox_{t-j} \right) + u_{3t}$$

In Eq. 4 both $rf\_ghg\text{-}co2$ and $co2$ are I(2), so this opens the possibility that they might themselves be cointegrated, with a linear combination of the two being I(1). However, this appears not to be the case, as the appropriate test statistic is just -2.04, well short of being significant. The equation to be estimated is then

$$g_t = \alpha_4 + \beta_{41} rf\_ghg - co2_t + \beta_{42} co2_t + \gamma_{41} solar_t + \gamma_{42} sox_t$$
$$+ \sum_{j=-p}^{p} \left(\phi_{41j}\Delta^2 rf\_ghg_{t-j} - co2_{t-j} + \phi_{42j}\Delta^2 co2_{t-j} + \phi_{43j}\Delta solar_{t-j} \right.$$
$$\left. + \phi_{44j}\Delta sox_{t-j} \right) + u_{4t}.$$

In these equations leads and lags of order p are included to ensure that any feedbacks from temperature to radiative forcings are taken into account. Such feedbacks are scientifically possible (see Kaufmann et al. 2006a) and their presence, if ignored, would lead to estimator inefficiency (see Saikkonen 1991; Stock and Watson 1993).

Table 3 presents DGLS estimates of each of the equations based on setting $p = 2$ (as suggested by Stock and Watson (1993), for annual samples of this size) and assuming that each error is generated as an AR(1) process. As the radiative forcings are successively aggregated (i.e., as we move from Eq. 4 through to Eq. 1), so the goodness of fit necessarily declines, but not markedly so. Indeed, it does not decline significantly, since the Wald statistics testing the hypothesis that all radiative forcing coefficients in a particular equation are equal are always insignificant. This is interesting, as the most disaggregated Eq. 4 has the most imprecisely estimated coefficients, with both $rf\_ghg\text{-}co2$ and $co2$ entering insignificantly (the two t-statistics having p-values of 0.21 and 0.71, respectively). However, joint tests on these two coefficients are able to reject the hypothesis that both are zero (with p-value less

Table 3 DGLS estimates of Eqs. 1–4 with standard errors shown in (); $ar(1)$ is the autoregressive parameter estimate

Equation no.	(1) *tot*	(2) *anth*	(3) *ghg*	(4) *ghg-co2*
a	−0.314 (0.021)	−0.303 (0.026)	−0.276 (0.055)	−0.257 (0.062)
$rf\_\#$	0.494 (0.051)	0.410 (0.110)	0.392 (0.117)	1.128 (0.900)
$co2$	–	–	–	0.148 (0.390)
$solar$	–	0.667 (0.245)	0.817 (0.368)	0.725 (0.386)
sox	–	–	0.424 (0.210)	0.582 (0.251)
$ar(1)$	0.518 (0.074)	0.518 (0.077)	0.549 (0.076)	0.576 (0.076)
$\overline{R}^2$	0.771	0.769	0.791	0.806
se	0.103	0.103	0.099	0.095
F	–	0.63 [0.43]	0.58 [0.56]	0.48 [0.70]

$\overline{R}^2$ is the coefficient of multiple determination adjusted for degrees of freedom. se is the residual standard error. F is the Wald statistic testing the hypothesis that all radiative forcing coefficients in an equation are equal; the marginal p-value is shown in [ ]
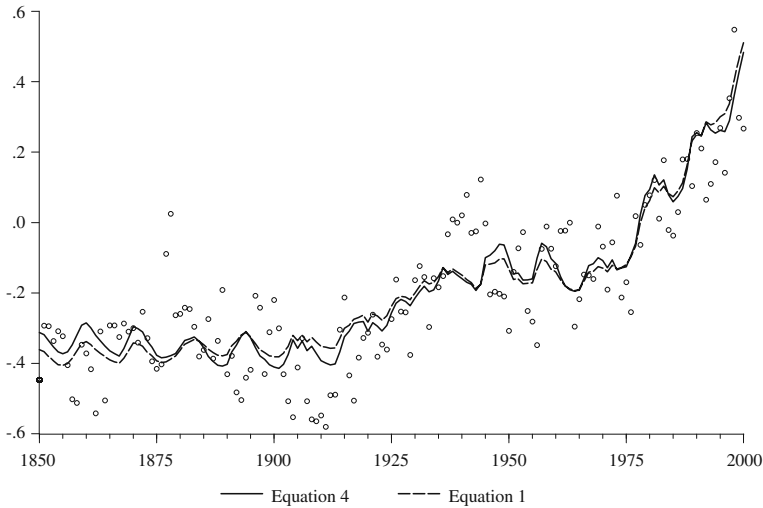
Fig. 4 Predicted temperature from cointegrating equations

than 0.001) but not that they are equal (*p*-value 0.44). Predicted temperatures from Eqs. 1 and 4 are shown in Fig. 4; note that these do not take into account the autoregressive error component, implicit in DGLS estimation, which would allow the short-run fluctuations in temperature to be modelled more accurately. The modest deterioration in fit achieved by aggregating from Eqs. 4 to 1 can be observed and this is consistent with the hypothesis tests reported above.

There is thus strong evidence that the hypothesis from physical theory that all forcings have an identical temperature effect is supported by the data. Consequently focusing on Eq. 1, a 95% confidence interval for β_1 is 0.494 ± 0.101. Following Kaufmann et al. (2006a, b), the temperature sensitivity to a doubling of radiative forcing and hence, equivalently, to a doubling of CO_2 concentration, is given by $6.3 \times 1n(2) \times \beta_1$°C. A 95% confidence interval for temperature sensitivity is thus 2.16 ± 0.44°C and this is consistent with Kaufmann et al. (2006a).

The autoregressive parameter is precisely estimated to be just above 0.5 in all regressions, consistent with the finding of cointegration. This implies that around 50% of any disequilibrium between radiative forcing and temperature is eliminated each year, which is a rate similar to that found by Kaufmann and Stern (2002) and Kaufmann et al. (2006a).

4 How robust is this temperature sensitivity estimate?

Given the importance of this temperature sensitivity result, it is necessary to examine its robustness. As it has been obtained from a cointegrating relationship fitted to the entire sample from 1850 to 2000, the robustness of the finding of cointegration should first be assessed. Within the testing framework used above, this can conveniently be done by focusing on Eq. 1 and considering the 'regime shift' extension

$$g_t = \alpha_1 + \gamma_1 \varphi_{t,\tau} + \beta_1 rf\_tot_t + \gamma_2 \left(rf\_tot_t \times \varphi_{t,\tau} \right) + u_{1t}. \tag{6}$$
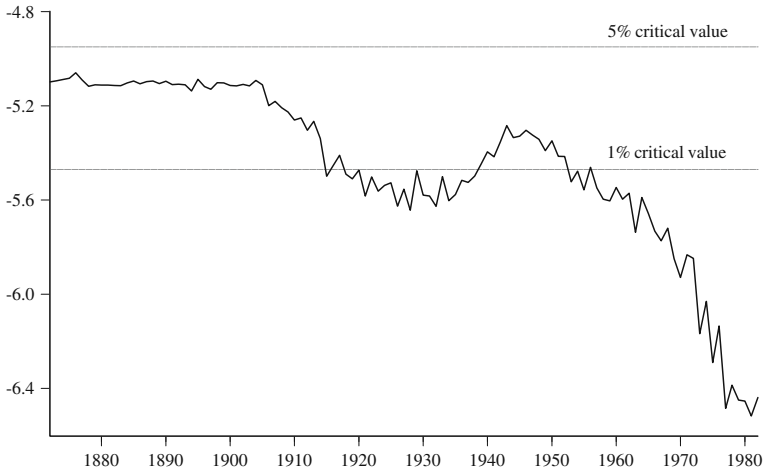
Fig. 5 Sequential cointegration test statistics from Eq. 1

The dummy variable $\varphi_{t,\tau}$ is defined as

$$\varphi_{t,\tau} = \begin{cases} 0 & \text{if } t \leq [\tau T] \\ 1 & \text{if } t \leq [\tau T] \end{cases}$$

where T (equal to 151 here) is the sample size, the unknown parameter $0 < \tau < 1$ denotes the relative timing of the shift, at which point the intercept and slope of the cointegrating relationship shift from α_1 and β_1 to $\alpha_1 + \gamma_1$ and $\beta_1 + \gamma_2$, respectively, and [ ] denotes integer part.
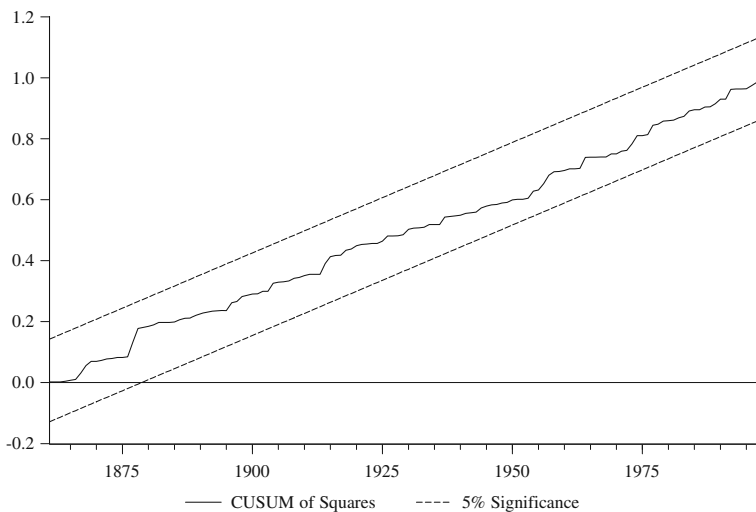

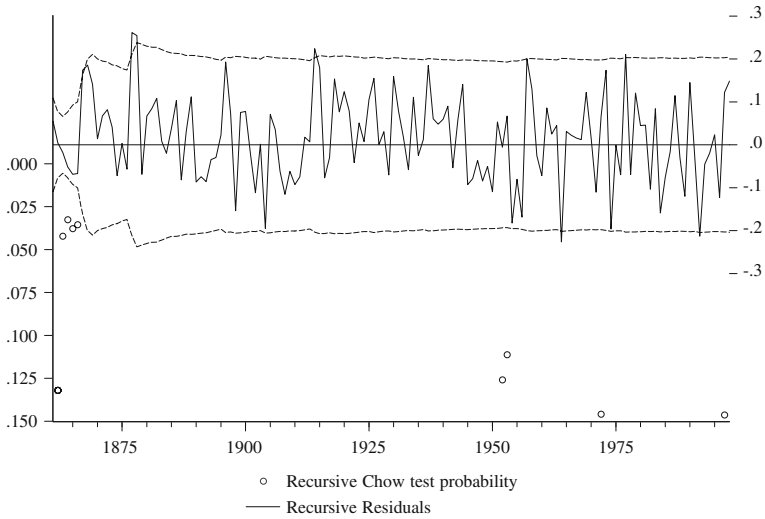**Fig. 6** CUSUM of squares plot from DFGLS estimation of Eq. 1

Fig. 7 Recursive residuals and Chow tests from DFGLS estimation of Eq. 1

Gregory and Hansen (1996) show that the null hypothesis of no cointegration can be tested in the face of a potential structural shift at an unknown point in time by estimating (6) sequentially across the set of break points $[\tau_1 T]$ to $[\tau_2 T]$ and calculating the sequence of ADF *t*-statistics from the analogous sequence of residual regressions (5). The test statistic is the minimum value (most negative) of the *t*-statistics, which may be compared with the critical values provided by Gregory and Hansen (1996; Table 1). Figure 5 shows the sequence of t-statistics obtained from setting $(\tau_1, \tau_2) = (0.15, 0.85)$, i.e., 1872 to 1982, which is a typical choice in these
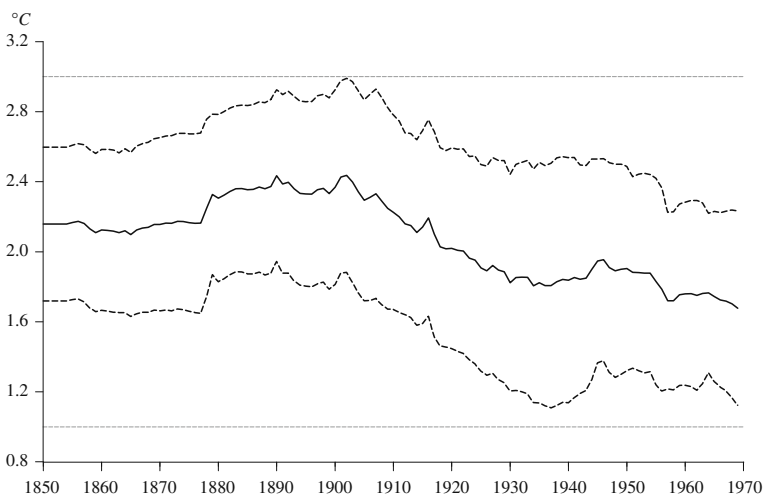

Fig. 8 Recursively estimated temperature sensitivity from Eq. 1 with 95% confidence bounds. *Horizontal lines* are at 1°C and 3°C

situations. Also shown are the 0.01 and 0.05 critical values of −4.95 and −5.47. The minimum value of −6.52 clearly rejects the null of no cointegration in the face of a possible structural shift and it is interesting that the entire sequence of test statistics breach at least the 0.05 critical value.

Given this strong and robust evidence of a cointegrating relationship irrespective of structural breaks, the stability of the relationship is now examined. Figure 6 shows the CUSUM of squares plot from the DFGLS regression of Eq. 1 (with a lagged dependent variable replacing the autoregressive error process). This is a standard test of model stability (Brown et al. 1975; Ploberger and Kramer 1992) and structural instability is indicated if the plot breaks the 95% confidence bands shown in the figure, which does not occur. Figure 7 shows the recursive residuals from this regression, along with sequential Chow (1960) tests for structural stability. No indication of instability is observed in the recursive residuals and only very early in the sample period is there any indication of structural shifts in the Chow tests.

Finally, Fig. 8 displays the sequence of temperature sensitivities, with accompanying 95% confidence intervals, obtained by estimating Eq. 1 'recursively', i.e., the horizontal axis shows the start year of each DGLS regression with the end year fixed at 2000 throughout. Thus the first point shows the temperature sensitivity estimated over the complete sample period, while the final point shows the sensitivity estimated over the last 30 years from 1970 to 2000. The estimates of temperature sensitivity are relatively stable, averaging 2.07°C and ranging from 1.68°C (for a sample starting at 1970) to 2.44°C (for a sample starting at 1902). The 95% confidence intervals are contained within a band of 1–3°C, thus giving a robust indication of the likely range of temperature sensitivity and one that is consistent with the robustness of the other statistics calculated above.

5 Conclusions

Using an updated data set of global temperature and radiative forcings, it has been shown that temperature and total radiative forcing cointegrate and that this relationship is stable across the period from 1850 to 2000. A robust estimate of the temperature sensitivity to a doubling of radiative forcing is calculated to be in the range of 1–3°C, with a point estimate of just over 2°C. Since we cannot reject the hypothesis that the different radiative forcings have an identical impact on temperature, this result can also be interpreted as providing the temperature sensitivity to a doubling of atmospheric CO_2 concentration. This is in line with the sensitivities calculated earlier by Kaufmann et al. (2006a) and Stern (2006) using related statistical methods and with the transient climate response simulated by climate models (Cubasch and Meehl 2001) and may be taken as providing further confirmation of the quantitative impact of CO2 forcing on temperatures.

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