HYDROLOGICAL PROCESSES
Hydrol. Process. **16**, 1807–1829 (2002)
Published online in Wiley InterScience (www.interscience.wiley.com). DOI: 10.1002/hyp.1095

The influence of autocorrelation on the ability to detect trend in hydrological series

Sheng Yue,[1]* Paul Pilon,[1] Bob Phinney[2] and George Cavadias[3]

[1] *Meteorological Services of Canada–Ontario Region, Environment of Canada, 867 Lakeshore Rd, PO Box 5050, Burlington, Ontario, L7R 4A6, Canada*
[2] *Ministry of the Environment, Guelph District 1, Stone Road W., Guelph, N1G 4Y2, Canada*
[3] *D61, 1321 Sherbrooke West, Montreal, Quebec, H3G 1J4, Canada*

Abstract:

This study investigated using Monte Carlo simulation the interaction between a linear trend and a lag-one autoregressive (AR(1)) process when both exist in a time series. Simulation experiments demonstrated that the existence of serial correlation alters the variance of the estimate of the Mann–Kendall (MK) statistic; and the presence of a trend alters the estimate of the magnitude of serial correlation. Furthermore, it was shown that removal of a positive serial correlation component from time series by pre-whitening resulted in a reduction in the magnitude of the existing trend; and the removal of a trend component from a time series as a first step prior to pre-whitening eliminates the influence of the trend on the serial correlation and does not seriously affect the estimate of the true AR(1). These results indicate that the commonly used pre-whitening procedure for eliminating the effect of serial correlation on the MK test leads to potentially inaccurate assessments of the significance of a trend; and certain procedures will be more appropriate for eliminating the impact of serial correlation on the MK test. In essence, it was advocated that a trend first be removed in a series prior to ascertaining the magnitude of serial correlation. This alternative approach and the previously existing approaches were employed to assess the significance of a trend in serially correlated annual mean and annual minimum streamflow data of some pristine river basins in Ontario, Canada. Results indicate that, with the previously existing procedures, researchers and practitioners may have incorrectly identified the possibility of significant trends.

KEY WORDS Mann–Kendall test; nonparametric test; time series analysis; trend analysis; serial correlation; pre-whitening, statistical hydrology

INTRODUCTION

In recent years, with growing concerns about the implications of green-house gases on the environment, researchers and practitioners have employed the nonparametric Mann–Kendall (MK) statistical test (Mann, 1945; Kendall, 1975) to identify whether monotonic trends exist in hydro-meteorological data such as temperature, precipitation, and streamflow. The advantage of the nonparametric statistical tests over the parametric tests, such as the *t*-test, is that the nonparametric tests are more suitable non-normally distributed, censored, and missing data, which are frequently encountered in hydrological time series (Hirsch and Slack, 1984). In fact, trend analyses are everyday occurrences in both theoretical and applied water resources studies. Examples include the studies of Hirsch and Slack (1984), Demaree and Nicolis (1990), Gan (1991, 1998), Gan and Kwong (1991), Chiew and McMahon (1993), Lettenmaier *et al.* (1994), Burn (1994), Yulianti and Burn (1998), Lins and Slack (1999), Douglas *et al.* (2000), Zhang *et al.* (2000, 2001), and others. The majority of these studies have assumed that observed data are serially independent. However, certain hydrological time series, such as annual mean and annual minimum streamflows, may frequently display statistically significant

* Correspondence to: Sheng Yue, Meteorological Services of Canada–Ontario Region, Environment of Canada, 867 Lakeshore Rd, PO Box 5050, Burlington, Ontario, L7R 4A6, Canada. E-mail: sheng.yue@ec.gc.ca

Received 23 October 2000
Accepted 21 January 2002

serial correlation. In such cases the existence of serial correlation will increase the probability that the MK test detects a significant trend (e.g. von Storch, 1995). This leads to a disproportionate rejection of the null hypothesis of no trend, whereas the null hypothesis is actually true.

In order to limit the influence of serial correlation on the MK test, pre-whitening was proposed by von Storch (1995) and Kulkarni and von Storch (1995). This procedure is intended to remove a serial correlation component such as a lag-one autoregressive (AR(1)) process from a time series. Zhang *et al.* (2000) employed this approach in detecting trends in Canadian temperature and precipitation records. In their work, the time series was assumed to consist of a linear trend ($T_t = bt$), an AR(1) process, and a noise component e_t. An iterative procedure was used for estimating the slope of the trend b and the lag-1 serial correlation r_1 of the models. This was done by first computing the lag-1 serial correlation coefficient of the time series and then removing the AR(1) component from the series. The slope of the trend was computed using the method by Theil (1950a–c) and Sen (1968), which hereinafter is referred to as the Theil–Sen approach (TSA). If the slope was found to be significant, the trend component was removed from the series. This procedure was repeated until the differences in the estimates of both b and r_1 in two consecutive iterations were smaller than a pre-set amount. Pre-whitening has also been applied to limit the influence of serial correlation on the MK test in the streamflow trend detection studies of Douglas *et al.* (2000), Zhang *et al.* (2001), Burn and Hag Elnur (2001), and others.

Another different method from the pre-whitening was proposed by Hamed and Rao (1998). They developed an empirical formula to compute effective sample size (ESS). The variance of the MK statistic was modified using the ESS to compensate for the effect of the serial correlation on the variance. This approach hereinafter is termed the variance correction approach (VCA), which is similar in spirit to the method proposed by Bayley and Hammersley (1946) and Lettenmaier (1976).

The above-reviewed two approaches dealing with the influence of serial correlation on the MK test do not address the potential interaction between a trend and an AR(1) process when both exist within a time series. The purpose of this study is to investigate this issue and to evaluate the ability of the existing procedures for reducing the effect of serial correlation on the MK test to detect a significant trend when a time series also comprises an autoregressive component. This involves observation of several issues: how the serial correlation affects the assessment of a significant trend by the MK test; how the trend affects the serial correlation; how the trend and the serial correlation influence each other when both exist in a time series; and how the removal of the trend and the AR(1) component affect the residual series. Later in this paper, a new approach is developed for more accurate assessment of the significance of a trend.

The remainder of the paper is organized as follows: in the second section we provide a brief description of the MK test; then we illustrate how the AR(1) process increases the probability of detecting trends in a time series when there is no trend. Following this we explore the influence of a trend on the serial correlation coefficient of a time series; we then investigate how an AR(1) process affects the trend; in the subsequent section we examine how the removal of either trend or AR(1) affects the residual series. Next we propose a new procedure for detecting a significant trend in serially correlated series, and in the penultimate section we apply the procedure to assess the significance of a trend in serially correlated annual mean and annual minimum daily streamflow at a number of streamflow monitoring sites in Ontario, Canada. The final section summarizes the results of the study.

THE MK TEST

The MK test, also called Kendall's tau test due to Mann (1945) and Kendall (1975), is the rank-based nonparametric test for assessing the significance of a trend, and has been widely used in hydrological trend detection studies. The null hypothesis H_0 is that a sample of data $\{X_i,\ i = 1, 2 \ldots, n\}$ is independent and identically distributed. The alternative hypothesis H_1 is that a monotonic trend exists in X. The statistic S of

Kendall's tau is defined as follows:

$$S = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \operatorname{sgn}(X_j - X_i) \tag{1}$$

where the X_j are the sequential data values, n is the length of the data set, and

$$\operatorname{sgn}(\theta) = \begin{cases} 1 & \text{if } \theta > 0 \\ 0 & \text{if } \theta = 0 \\ -1 & \text{if } \theta < 0 \end{cases} \tag{2}$$

Mann (1945) and Kendall (1975) have documented that when $n \geq 8$, the statistic S is approximately normally distributed with the mean and the variance as follows:

$$E(S) = 0 \tag{3}$$

$$V(S) = \frac{n(n-1)(2n+5) - \sum_{m=1}^{n} t_m m(m-1)(2m+5)}{18} \tag{4}$$

where t_m is the number of ties of extent m. The standardized test statistic Z is computed by

$$Z = \begin{cases} \dfrac{S-1}{\sqrt{V(S)}} & S > 0 \\ 0 & S = 0 \\ \dfrac{S+1}{\sqrt{V(S)}} & S < 0 \end{cases} \tag{5}$$

The standardized MK statistic Z follows the standard normal distribution with mean of zero and variance of one.

The probability value P of the MK statistic S of sample data can be estimated using the normal cumulative distribution function as

$$P = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-t^2/2}\, dt \tag{6}$$

For independent sample data without trend the P value should be equal to 0·5. For sample data with a large positive trend the P value should be closer to 1·0, whereas a large negative trend should yield P value closer to 0·0.

THE EFFECT OF THE AR(1) PROCESS ON TYPE I ERROR

Von Storch (1995) demonstrated that the existence of positive serial correlation in a time series increases the probability that the MK test will detect a significant trend, i.e. serial correlation increases the type I error. To verify von Storch's finding, we generate various series with different lag-1 serial correlation coefficients ρ_1 using Monte Carlo simulation as follows:

$$X_t = \mu_X + \rho_1(X_{t-1} - \mu_X) + \varepsilon_t \tag{7}$$

where μ_X is the mean of X_t, ε_t is the white-noise process with mean $\mu_\varepsilon = 0$ and variance $\sigma_\varepsilon^2 = \sigma_X^2(1 - \rho_1^2)$ in which σ_X^2 is the variance of X_t. Given that ξ_t is a normally distributed random variable with mean $\mu_\xi = 0$ and variance $\sigma_\xi^2 = 1$, then Equation (7) can be rewritten as

$$X_t = \mu_X + \rho_1(X_{t-1} - \mu_X) + \sigma_X\sqrt{1 - \rho_1^2}\,\xi_t \tag{8}$$

In practice, we replace the mean μ_X, variance σ_X^2, and lag-one serial correlation coefficient ρ_1 by the sample's mean $E(X)$, variance $V(X)$, and lag-1 serial correlation coefficient r_1, respectively.

The simulation generated 1000 time series of the AR(1) processes using Equation (8) for each sample size ($n = 20$, 50, 100, 150) with different given $\rho_1 = 0$ (0·1) 0·9 for $X_0 = \mu_X$, $\mu_X = 1{\cdot}0$ and $\sigma_X^2 = 0{\cdot}5$.

AR(1) with positive serial correlation

For each generated time series, with given sample size n and lag-1 serial correlation ρ_1, the MK statistics S_i ($i = 1$ (1) 1000) can be computed using Equation (1). The confidence interval of the MK statistic of the two-tailed test can be estimated by

$$S \in \left[z_{\alpha/2}\sqrt{V(S)}, z_{1-\alpha/2}\sqrt{V(S)}\right] \tag{9a}$$

where α is the pre-assigned significance level, $z_{\alpha/2}$ and $z_{1-\alpha/2}$ are respectively the $\alpha/2$ and $1 - \alpha/2$ quantiles of the standard normal distribution. For an $\alpha = 0{\cdot}05$, $z_{\alpha/2}$ and $z_{1-\alpha/2}$ are equal to $-1{\cdot}96$ and $1{\cdot}96$ respectively.

The sample variance $V(S)$ of the MK statistic is calculated using Equation (4). Correspondingly, the critical or rejection regions of the MK statistic S can be approximately given by

$$S < z_{\alpha/2}\sqrt{V(S)} \quad \text{or} \quad S > z_{1-\alpha/2}\sqrt{V(S)} \tag{9b}$$

Given that the series is generated without trend, the type I error or the rejection rate can be computed as

$$\mathrm{ERI} = \frac{N_{\mathrm{rej}}}{N} \tag{10}$$

in which N_{rej} is the total number of the computed MK statistics falling in the critical regions and N is the number of simulations ($N = 1000$). From statistical inference theory, it is expected that 50 out of the 1000 S values should fall in the rejection regions. This is indicated by the first four bars corresponding to $\rho_1 = 0$ in Figure 1a. It can be seen that as the lag-1 serial correlation increases, the type I error ERI also increases. There is an increasing tendency to reject the null hypothesis of no trend, whereas in fact it is true, as the serial correlation increases. From Figure 1a, it can also be seen that the ERI is not sensitive to the sample sizes selected for this experiment. These simulation results concur with those obtained by von Storch (1995).

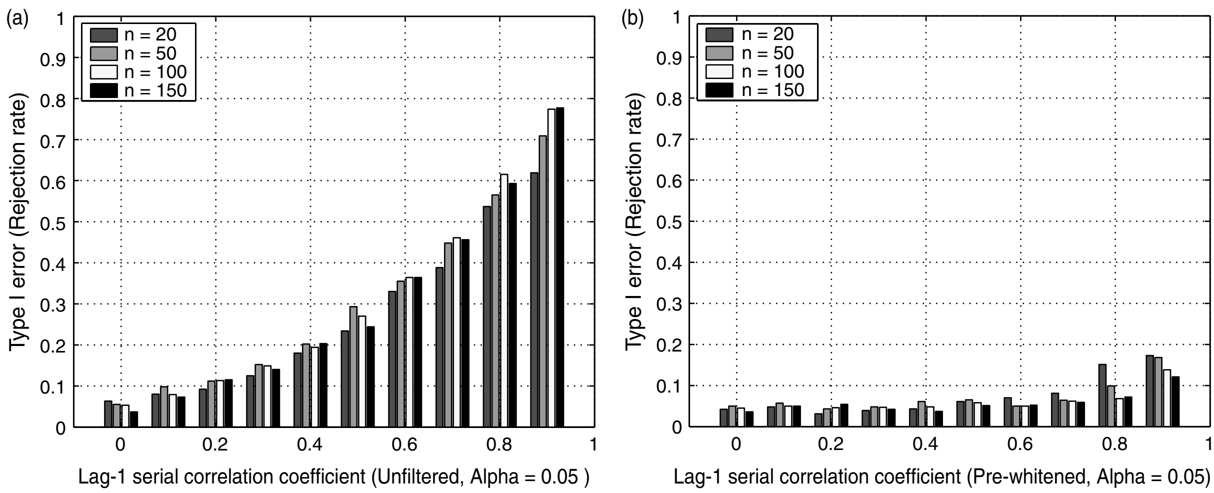

Figure 1. Effect of positive serial correlation on the type I error: (a) unfiltered; (b) pre-whitened

Removal of AR(1) component by pre-whitening

Pre-whitening has been suggested by von Storch (1995) and has been used by Kulkarni and von Storch (1995), Douglas *et al.* (2000), Zhang *et al.* (2000, 2001), and Burn and Hag Elnur (2001) to reduce the influence of an AR(1) component on the application of the MK test. The series can be pre-whitened using the following formula:

$$Y_t = X_t - r_1 X_{t-1} \tag{11}$$

The type I errors can be estimated from the residual time series Y_t, using the inequalities in Equation (9b) and Equation (10). Figure 1b demonstrates that these type I errors are almost the same as the nominal significance level, i.e. the serial correlation has effectively been removed from the series.

AR(1) with negative serial correlation

The preceding subsection explored the influence of the presence of positive serial correlation in the time series on the MK test. Following the same procedure used in the section, the impact of the presence of negative serial correlation on the results of the MK test was also examined. The type I errors of the generated series corresponding to given negative serial correlation $\rho_1 = 0\ (-0{\cdot}1)\ -0{\cdot}9$ are illustrated in Figure 2a. In contrast to the results obtained in 'AR(1) with positive serial correlation', the influence of the negative autocorrelation on the MK test may tend to underestimate the probability of detecting trends. This results in an increase in the type II error. The type I errors of the corresponding pre-whitened series are displayed in Figure 2b. Similar to the results of the positive case, the type I errors return to near the nominal significance level of 0·05. Hydrological time series such as rainfall and streamflow more frequently display positive persistence (positively serially correlated) than negative. The main attention in this paper is focused on the influence of positive autocorrelation on the MK test.

How does the existence of serial correlation influence the rejection rate of the null hypothesis of no trends for the MK test?

It is meaningful to know why the existence of positive serial correlation can cause an increase in the false rejection rate of the null hypothesis of no trends for the MK test. Figure 3 shows the fitted probability density functions of the MK statistics of the simulated series corresponding to the sample size $n = 100$ given in Figure 1a. It is apparent that the existence of positive serial correlation in a time series does not alter the

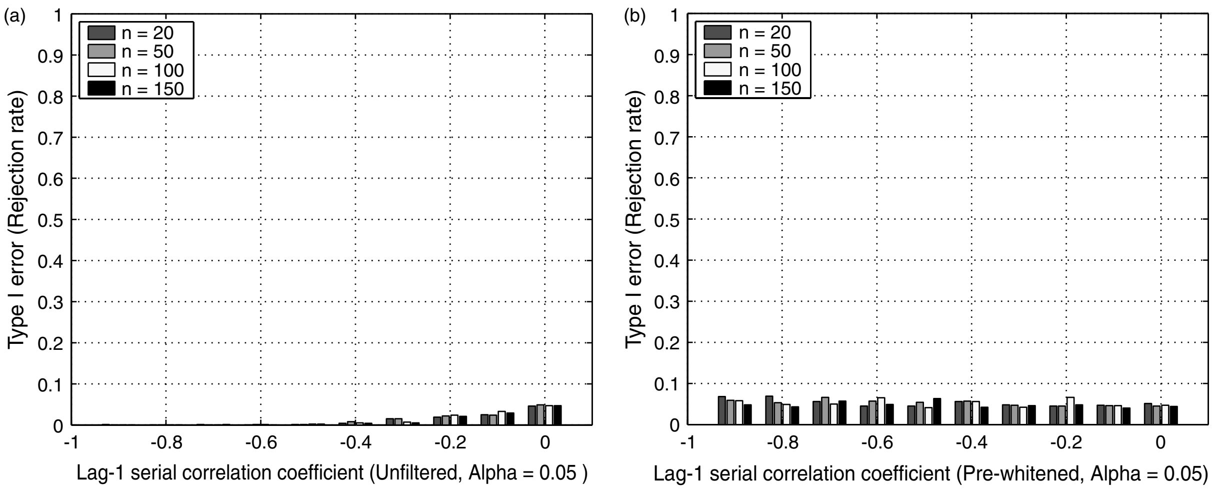

Figure 2. Effect of negative serial correlation on the type I error: (a) unfiltered; (b) pre-whitened

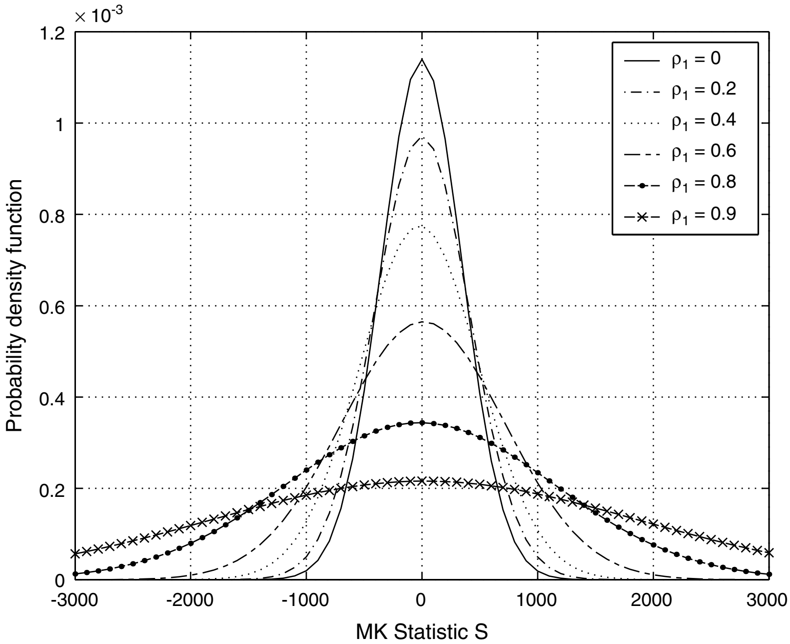

Figure 3. Effect of positive serial correlation on the M K statistic

asymptotic normality of the MK statistic S, nor does it alter the location of the centre of the distribution or the mean of S. It is evident that the presence of positive serial correlation changes the dispersion of the distribution. The variance of S increases as the magnitude of serial correlation increases. For a time series with negative serial correlation, opposite to the positive case, the existence of negative serial correlation decreases the variance of the MK statistic (see Figure 4). These diagrams provide empirical evidence for the VCA proposed by Hamed and Rao (1998).

Applicability of the VCA

Hamed and Rao (1998) proposed correcting the variance of S by using an effective sample size (ESS) that reflects the effect of serial correlation on the variance of S, as illustrated in the previous section. The modified variance is given by

$$V^*(S) = V(S)\frac{n}{n^*} \tag{12a}$$

where $V(S)$ is the variance of the MK statistic S for the original sample data, estimated using Equation (4); n is the sample size; n^* is the ESS; and n/n^* is the correction factor due to the existence of the serial correlation in sample data.

From Equation (5), the modified standardized MK statistic Z^* is given by

$$Z^* = \begin{cases} \dfrac{S-1}{\sqrt{V^*(S)}} & S > 0 \\ 0 & S = 0 \\ \dfrac{S+1}{\sqrt{V^*(S)}} & S < 0 \end{cases} \tag{12b}$$

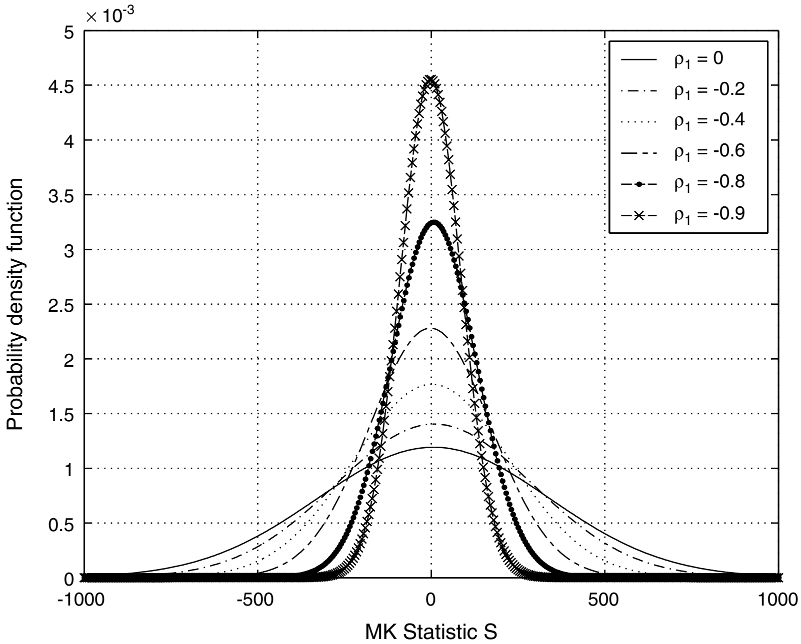

Figure 4. Effect of negative serial correlation on the M K statistic

The correction factor is computed by

$$n/n^* = 1 + \frac{2}{n(n-1)(n-2)} \sum_{j=1}^{n-1} (n-k)(n-k-1)(n-k-2) r_k^R \tag{13}$$

where r_k^R is the lag-k serial correlation coefficient of the ranks RX_t of the sample data X_t. r_k^R is computed by replacing the sample data X_t by their ranks RX_t in the following formula (Salas *et al.*, 1980):

$$r_k = \frac{\dfrac{1}{n-k} \sum_{t=1}^{n-k} [X_t - E(X_t)][X_{t+k} - E(X_t)]}{\dfrac{1}{n} \sum_{t=1}^{n} [X_t - E(X_t)]^2} \tag{14a}$$

$$E(X_t) = \frac{1}{n} \sum_{t=1}^{n} X_t \tag{14b}$$

where r_k is the lag-k serial correlation coefficient of the sample data X_t, and $E(X_t)$ is the mean of the sample data.

Hamed and Rao (1998) have suggested that only the significant r_k^R values be taken into account. For a significance level of 0·05, the correction factor n/n^* was computed. The corrected variance $V^*(S)$ was then used to replace the variance $V(S)$ in the inequalities of Equations (9a) and (9b) for reconstruction of the confidence intervals and the critical regions. The type I errors for the generated time series were recomputed by Equation (10), and are illustrated in Figure 5. It can be seen that, for the most part, the false rejection rate has been decreased compared with the classical MK test results of Figure 1a. They are, however, still much higher than the pre-assigned significance level for $n = 20$ and 50. In addition, for $n = 150$ and $0 < \rho_1 < 0{\cdot}8$,

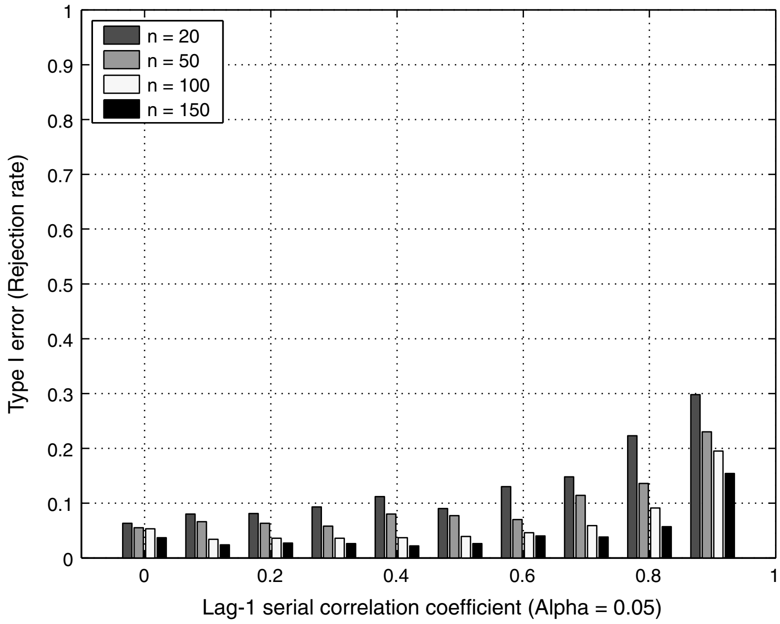

Figure 5. Effect of positive serial correlation on the type I error by the VCA

the rejection rate is slightly less than the significance level of 0·05. The study of Thiebaux and Zwiers (1984) when investigating the properties of a similar approach have demonstrated that it is difficult to estimate effective sample size reliably.

THE EFFECT OF A TREND ON SERIAL CORRELATION

The purpose of trend detection studies by statistical tests such as the MK test is to assess if there is a significant trend in a time series. In the preceding section, the simulation experiments were devised to explore the effect of the presence of an AR(1) process on the type I error by the MK test in the case that the generated time series only consisted of an AR(1) process and a noise. However, if a trend did exist in a time series, even though the time series did not comprise an AR(1) process, there would possibly be some significant serial correlation. This seemly 'false' detection of an AR(1) is produced by the presence of a trend, thereby resulting in a misinterpretation that the series includes an AR(1) process. It is important to know how and to what degree a trend will contaminate the estimate of serial correlation. This issue is investigated through superimposing a linear trend onto a random time series generated without an AR(1) process.

In order to explore how much a trend component will contribute to the autocorrelation of the whole series, a time series is generated consisting of two components comprising trend T_t and randomness e_t. First, normally distributed random time series are generated with sample sizes $n = 30$, 40, 50, 60, 80, 100, mean $E(e_t) = 1{\cdot}0$, and variance $V(e_t) = (0{\cdot}1i)^2$ ($i = 1$ (1) 10). The corresponding standard deviations and coefficients of variations $C_V = \sqrt{V(e_t)}/E(e_t)$ for the experiment are 0·1 (0·1) 1·0. Second, a trend component is superimposed onto the random time series with slope $\beta = 0{\cdot}002$ (0·002) 0·010. For a time series with a sample size of 100 and a mean $E(e_t) = 1{\cdot}0$, the mean value of the series would correspond to increases of 20%, 40%, 60%, 80%, and 100% respectively over 100 years for each of the generated random time series (see Appendix A). The lag-1 serial correlation coefficient r_1 is then estimated using Equation (14a), for each of the composite series for various β and C_V combinations.

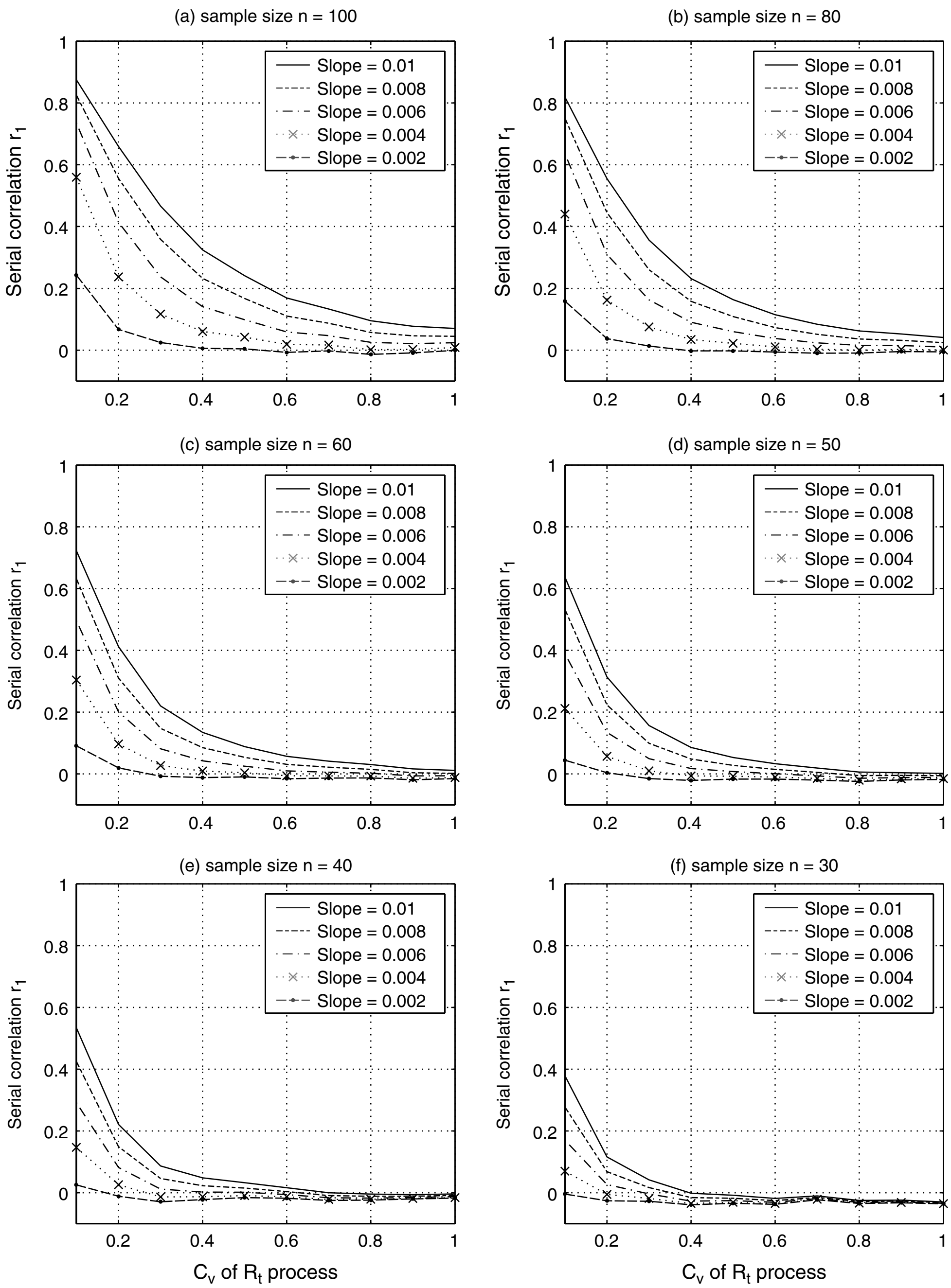

Figure 6. Influence of trend on serial correlation; serial correlation versus the coefficient of variance of the random component curve corresponding to various trends

The results for the sample size $n = 30$, 40, 50, 60, 80, 100 are displayed in Figure 6a–f. As the coefficients of variations C_V of the random component e_t increase, the estimated sample lag-1 serial correlation coefficient decreases. The lag-1 serial correlation coefficient shows a decreasing influence of a trend on its magnitude as the C_V increases.

The above results can also be derived analytically as follows. Let

$$Y_t = \beta t + e'_t \tag{15}$$

where e'_t is the random variable with mean $E(e'_t) = 0$ and variance $V(e'_t)$. To simplify the computations and without loss of generality, assume $E(t) = 0$, and consequently $E(Y_t) = 0$. On the basis of Equation (14a), the lag-1 serial correlation coefficient r_1 is given by

$$r_1 = \frac{\sum [Y_t - E(Y_t)][Y_{t-1} - E(Y_t)]}{\sum [(Y_t - E(Y_t)]^2} = \frac{\sum (\beta t + e'_t)[\beta(t-1) + e'_{t-1}]}{\sum (\beta t + e'_t)^2} \tag{16a}$$

The random variable e'_t is uncorrelated with t and e'_{t-1}; therefore, the above expression can be written as:

$$r_1 = \frac{\beta^2 \sum t^2}{\beta^2 \sum t^2 + \sum e'^2_t} = \frac{1}{1 + \dfrac{n-1}{\sum t^2} \dfrac{V(e'_t)}{\beta^2}} = \frac{1}{1 + \eta \dfrac{V(e'_t)}{\beta^2}} \tag{16b}$$

For a given sample size n, η is constant. An inspection of this formula confirms the conclusions drawn from the above simulation experiments.

THE INFLUENCE OF AN AR(1) PROCESS ON TREND

The section entitled 'The effect of the AR(I) process on type I error' demonstrated the influence of an AR(1) process on the MK statistic. In this section, the same approach is followed, but interest is focused on the assessment of the impact of an AR(1) process on the estimate of the magnitude of the slope of a trend. The slope of a trend is estimated using the TSA (Theil, 1950a–c; Sen, 1968), and it is estimated as follows:

$$b = \text{Median}\left(\frac{X_j - X_l}{j - l}\right) \forall l < j \tag{17}$$

where b is the estimate of the slope of the trend and X_l is the lth observation. The slope determined by the TSA is a robust estimate of the magnitude of a trend. Since the appearance of the paper of Hirsch *et al.* (1982), the TSA has been popularly employed for identifying the slope of trends in hydrological time series (e.g. Hirsch and Slack, 1984; Demaree and Nicolis, 1990; Gan, 1991, 1998; Gan and Kwong, 1991; Lettenmaier *et al.*, 1994; Burn, 1994; Yulianti and Burn, 1998; and others).

This experiment comprises 1000 simulations with sample size 100 and lag-1 serial correlation $\rho_1 = 0{\cdot}0$, 0·2, 0·4, 0·6, 0·8, 0·9. No trend is superimposed on the series. The fitted probability density functions of the estimated slopes corresponding to given ρ_1 are plotted in Figure 7. It can be seen that the impact of the AR(1) process on the estimate of the slope is almost identical to that on the MK statistic S. The existence of a positive AR(1) process alters the variance of the estimate of the slope, whereas it does not change the centre (mean) and the distribution type of the slope of trend.

PERFORMANCE OF A LINEAR TREND, AN AR(1) PROCESS, AND A NOISE

The preceding sections have discussed how a trend affects the serial correlation in the case that a series only consists of a linear trend and a random variable. They also cover how an AR(1) process influences the estimate of the variance of trend in the case that a series only consists of an AR(1) process and noise. However, in practice, one can only sample the lag-1 serial correlation coefficient estimated by Equation (14a) and the slope

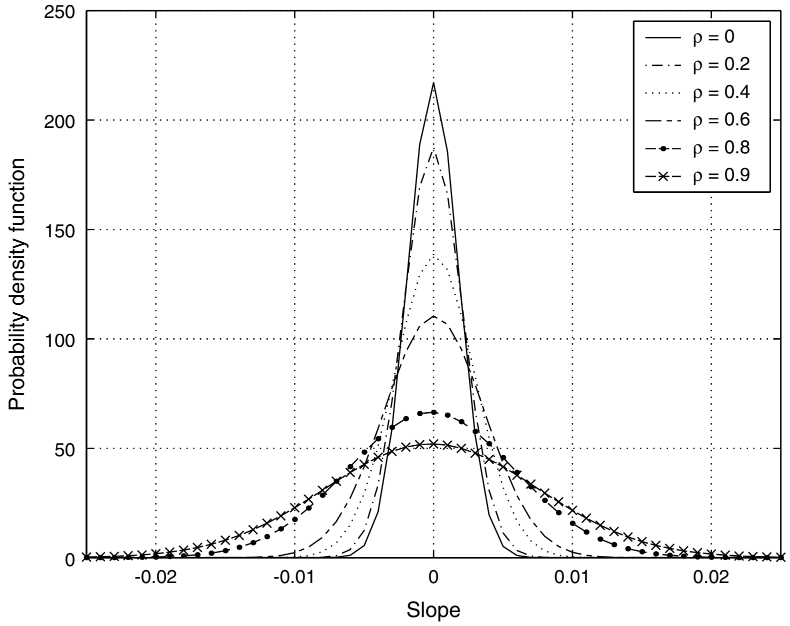

Figure 7. Influence of the AR(1) process on the slope of trend

of a trend by Equation (17) based on sample data. The previous analyses have documented that the trend and serial correlation components interact, and it is not known how much each may be contributing to the other. This section documents some results of Monte Carlo experiments designed to provide some knowledge on how these components may be interacting. The experiment blends a linear trend and an AR(1) process with noise and explores the performance of the blended series. Each component is, in turn, separated from the generated series, in order to establish its effect on the residual series.

Following the generation procedure described in the section entitled 'The effect of the AR(I) process on type I error', 1000 AR(1) series were generated with sample size $n = 100$, mean of 1·0, and standard deviation of 0·5, and with lag-1 serial correlation $\rho_1 = 0{\cdot}2\ (0{\cdot}2)\ 0{\cdot}8$. Then a trend series having slope $\beta = 0{\cdot}002\ (0{\cdot}002)$ 0·008 was superimposed on each of the AR(1) series.

The effect of removing the trend and the AR(1) on the serial correlation of the remaining series

The lag-1 serial correlation coefficient r_1 and slope b of each combined series were estimated using Equations (14a) and (17) respectively. The trend was then removed from the combined series X_t by

$$Y_t^{\mathrm{RT}} = X_t - bt \tag{18}$$

to form a 'trend-removed' residual series. The combined series was also pre-whitened using Equation (11) with the estimated serial correlation r_1, and this is referred to as the 'pre-whitened' residual series. The lag-1 serial correlations of these two residual series were recomputed. The histograms and fitted normal curves of the lag-1 serial correlation coefficients of the combined series, the trend-removed series, and the pre-whitened series are displayed in the first, second, and third rows of Figures 8 through 11, which correspond to the different true slopes $\beta = 0{\cdot}002\ (0{\cdot}002)\ 0{\cdot}008$ respectively. In these diagrams, the first, second, third, and fourth columns represent the series corresponding to the true AR(1): $\rho_1 = 0{\cdot}2\ (0{\cdot}2)\ 0{\cdot}8$ respectively. For ease of comparison, the population lag-1 serial correlation and the mean of the estimated lag-1 serial

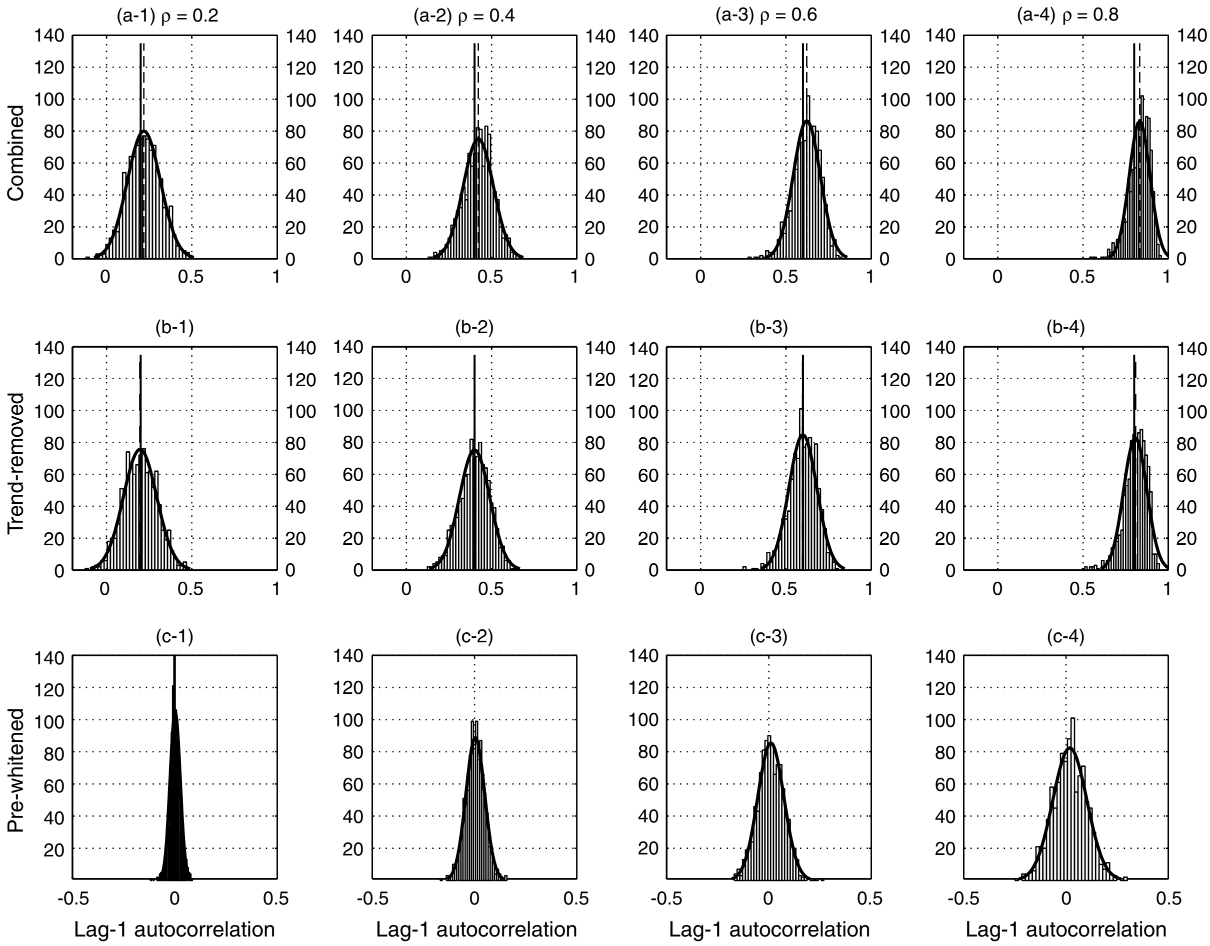

Figure 8. Serial correlation of the combined series, trend-removed series, and pre-whitened series (slope $\beta = 0.002$)

correlation coefficient of the three above series are displayed by vertical solid and dashed lines respectively in the diagrams in the first and second rows of Figures 8 through 11.

By viewing the first rows in Figures 8 through 11, it is evident that the means of the estimated correlation coefficients of the combined series are greater than the true or parent serial correlation. The differences between the population serial correlation coefficient ρ_1 and the means of the estimated correlation coefficient r_1 from the combined series decrease slightly as the population correlation increases for the large β values. Also, the differences between the population serial correlations and the means of the estimated correlation coefficients of the combined series increase with an increase in the true trend of the series. This further confirms our findings as presented in the section entitled 'The effect of a trend on serial correlation'.

From the second row of diagrams in Figures 8 through 11, it can be seen that the means of the estimated lag-1 serial correlations of all the trend-removed series are almost the same as the population serial correlation. The influence of trend on the estimate of the serial correlation has been nearly eliminated once the trend component has been removed.

The diagrams in the third rows of Figures 8 through 11 indicate that the means of the estimated serial correlations of all of the pre-whitened series are almost equal to zero. The pre-whitening process has effectively

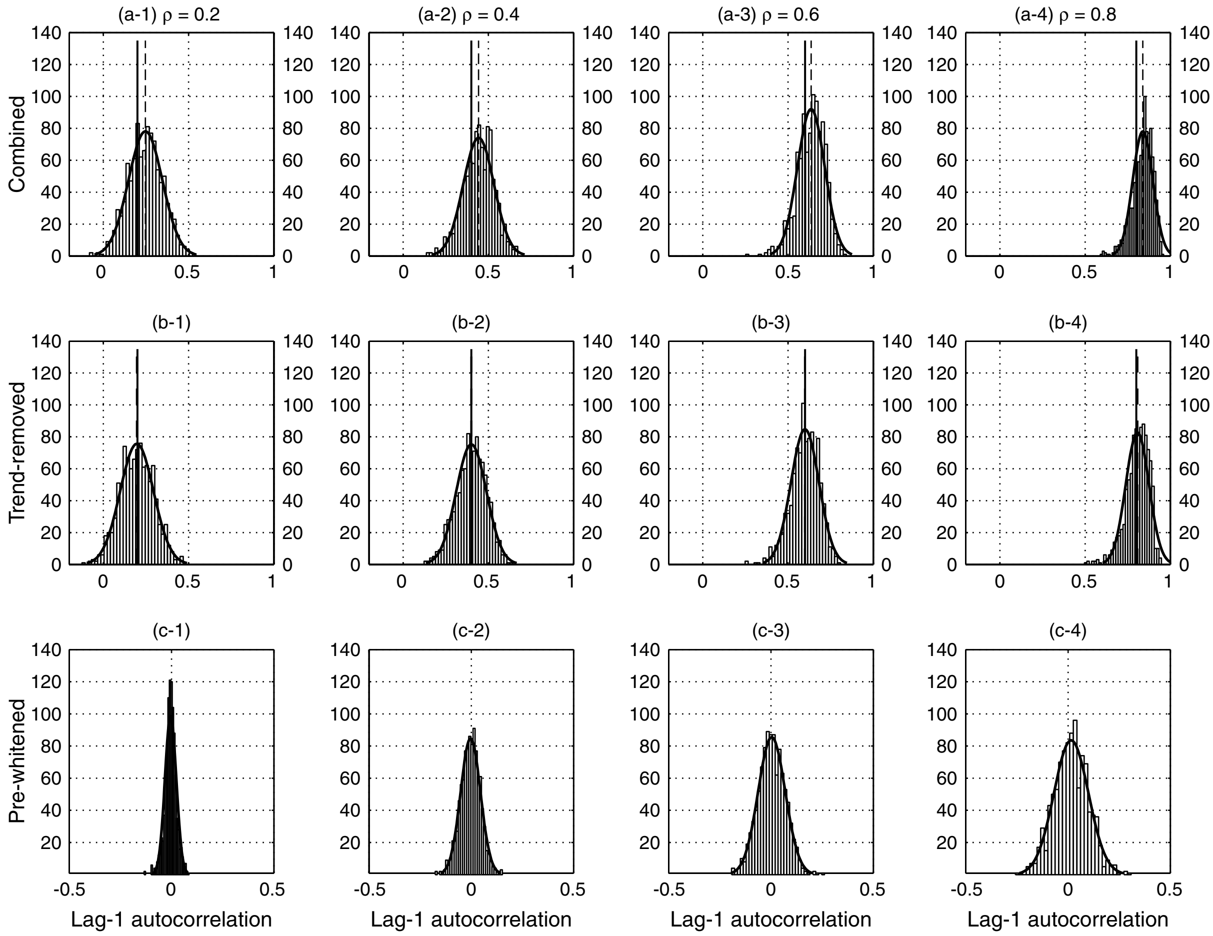

Figure 9. Serial correlation of the combined series, trend-removed series, and pre-whitened series (slope $\beta = 0.004$)

removed the autocorrelation from the combined series. The pre-whitened residual series no longer displays the existence of an AR(1) process.

The effect of removing the AR(1) and trend on the magnitude of trend of the residual series

The slopes of the combined series, the pre-whitened series, and the trend-removed series as estimated by Equation (17) are presented in the first, second, and third rows of Figures 12 through 15. Each of these figures corresponds to the population lag-1 serial correlations $\rho_1 = 0{\cdot}2\ (0{\cdot}2)\ 0{\cdot}8$. The first, second, third, and fourth columns in these diagrams represent the series corresponding to the different slopes $\beta = 0{\cdot}002\ (0{\cdot}002)$ $0{\cdot}008$ respectively. Again, for ease of comparison, these slopes and the means of the estimated slopes of the combined series and the pre-whitened series are shown by the vertical solid and dashed lines respectively. These lines appear in the diagrams in the first and second rows of Figures 12 through 15.

The first rows of Figures 12 through 15 demonstrate that the existence of serial correlation does not alter the mean estimate of the slopes of the trend, whereas the variance of the estimate of the slopes increases as serial correlation increases. This was observed in the section entitled 'The influence of an AR(1) process on trend'.

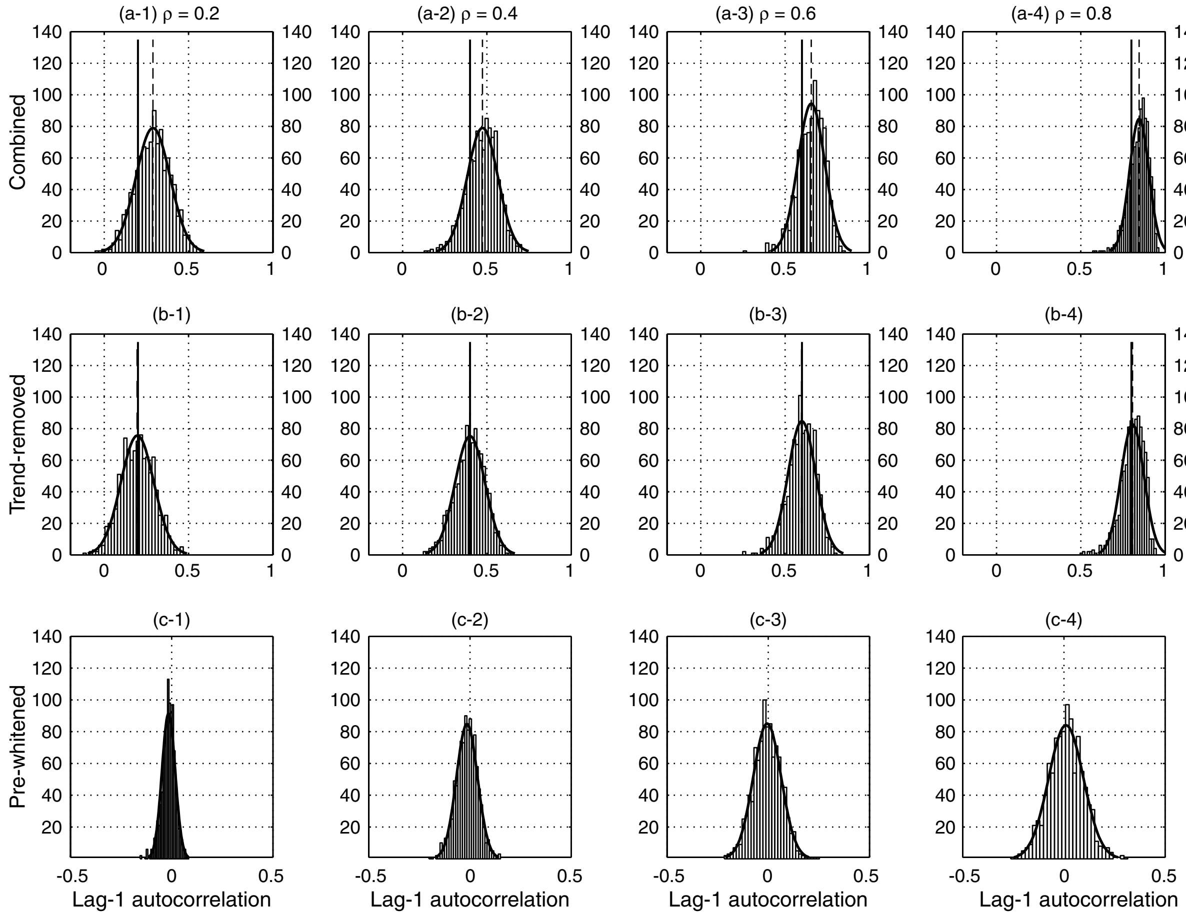

Figure 10. Serial correlation of the combined series, trend-removed series, and pre-whitened series (slope $\beta = 0.006$)

The mean of the estimated slopes of the pre-whitened series are less than the population slopes, especially for higher β and higher ρ_1 values. These results are depicted in the graphs of the second row of Figures 12 through 15. This is partially due to the influence of trend on the estimate of the serial correlation. In essence, the existence of trend inflates the estimate of the serial correlation beyond the population value. The first row of graphs within Figures 8 through 11 also illustrates this point. The removal of the AR(1) using the inflated estimate of the serial correlation coefficient r_1 leads to a loss or a compensatory reduction in the estimate of the slope of the trend, dragging it below the population value.

The third rows of graphs within Figures 12 through 15 indicate that the slopes of all the trend-removed series are equal to zero. This is evidence that the trend component can be effectively removed using Equation (18), provided it is accomplished prior to removal of the AR(1) process.

From the above investigation, the following observations can be drawn. The pre-whitening approach does remove the AR(1) component, but it also removes part of the magnitude (slope) of the trend compared with the true one. After pre-whitening, the estimate of the slope becomes much smaller than it was prior to the pre-whitening. Removal of positive serial correlation by pre-whitening as a first step in the overall process appears to underestimate the magnitude of the slope and its statistical significance. Furthermore, altering the sequence of the steps within the processes such that trend is removed as a first step may allow for a more accurate estimate of the population's AR(1) process and a subsequent better estimation of the significance of trend.

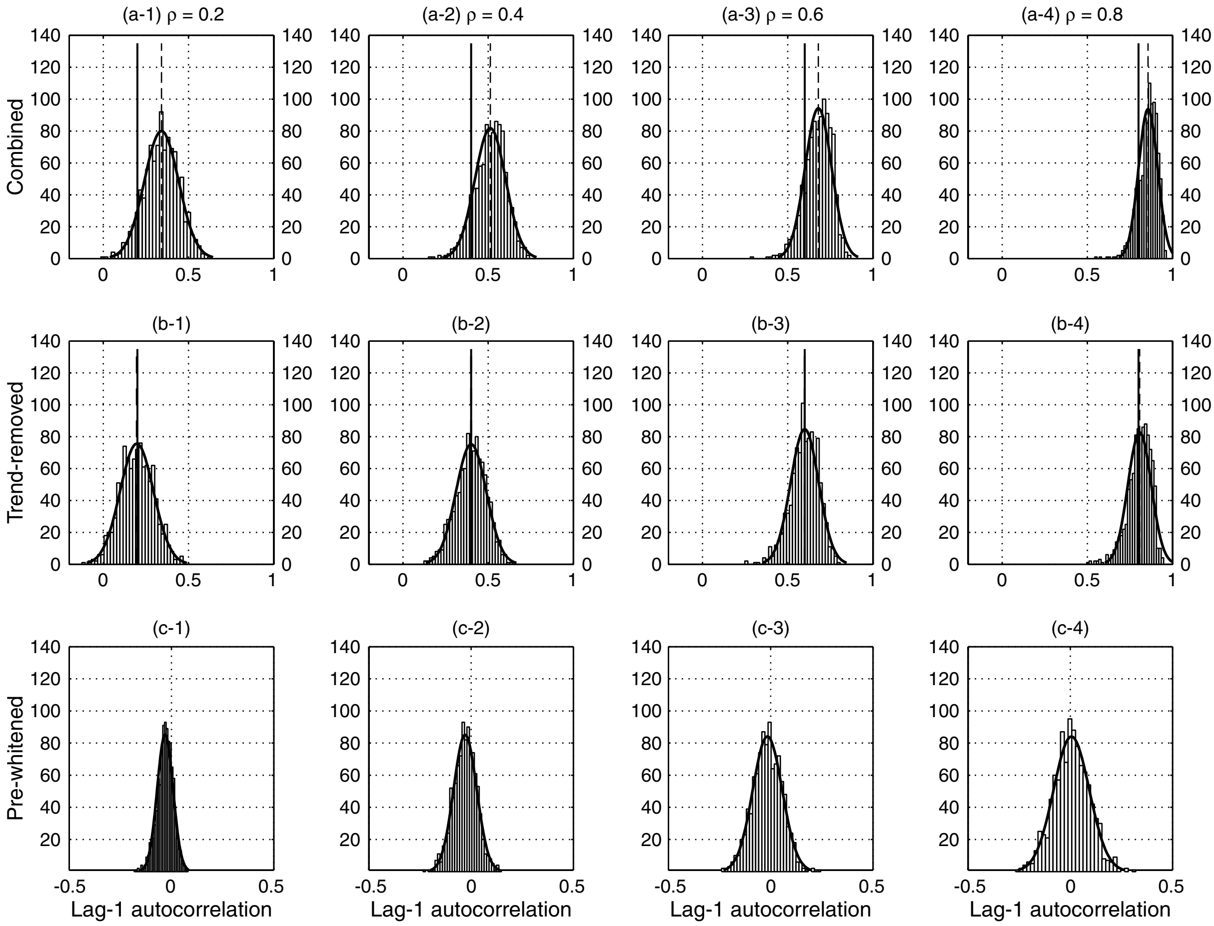

Figure 11. Serial correlation of the combined series, trend-removed series, and pre-whitened series (slope $\beta = 0.008$)

Classical time series analysis is based on the assumption of stationarity. In the case that a deterministic component such as a trend exists, standard statistical text books on time series analysis (e.g. see Salas *et al.* (1980), Salas (1993), Kendall and Ord (1990), and Wei (1990)) suggest that, before constructing the appropriate stochastic model for representing a time series, the deterministic trend component be removed first from the time series by an approach such as parametric regression analysis. Prior literature does not provide a rationale for this order of treatment. However, these new simulation experiments give empirical evidence for the validity of removing the trend component first.

A PROPOSED PROCEDURE FOR DETECTING TRENDS IN SIGNIFICANTLY SERIALLY CORRELATED HYDROLOGICAL SERIES

On the basis of the simulation results obtained in the preceding section, the following procedure is proposed to detect a significant trend in a time series with significant serial correlation.

Step 1. The slope b of a trend in sample data is estimated by the TSA. If the slope is almost equal to zero, then it is not necessary to continue to conduct trend analysis. If it differs from zero, then it is assumed

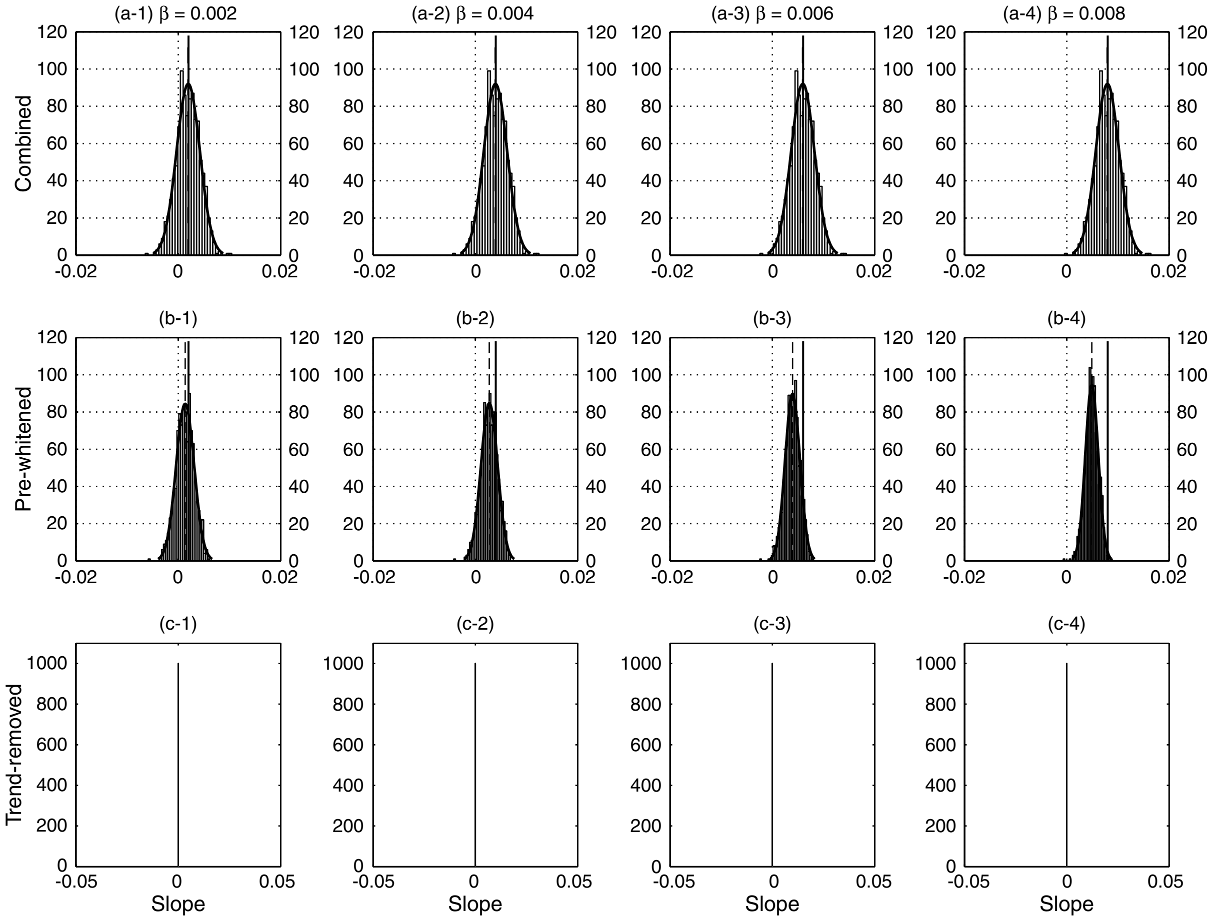

Figure 12. Slope of the trend of the combined series, pre-whitened series, and trend-removed series ($\rho_1 = 0.2$)

to be linear, and the sample data are detrended by

$$X'_t = X_t - T_t = X_t - bt \tag{19}$$

Step 2. The lag-1 serial correlation coefficient r_1 of the detrended series X'_t is computed using Equation (14a) and then the AR(1) is removed from the X'_t by

$$Y'_t = X'_t - r_1 X'_{t-1} \tag{20}$$

This pre-whitening procedure after detrending the series is referred to as the trend-free pre-whitening (TFPW) procedure. The residual series after applying the TFPW procedure should be an independent series.

Step 3. Third, the identified trend T_t and the residual Y'_t are blended by

$$Y_t = Y'_t + T_t \tag{21}$$

It is evident that the blended series Y_t could preserve the true trend and is no longer influenced by the effects of autocorrelation.

Step 4. The MK test is applied to the blended series to assess the significance of the trend.

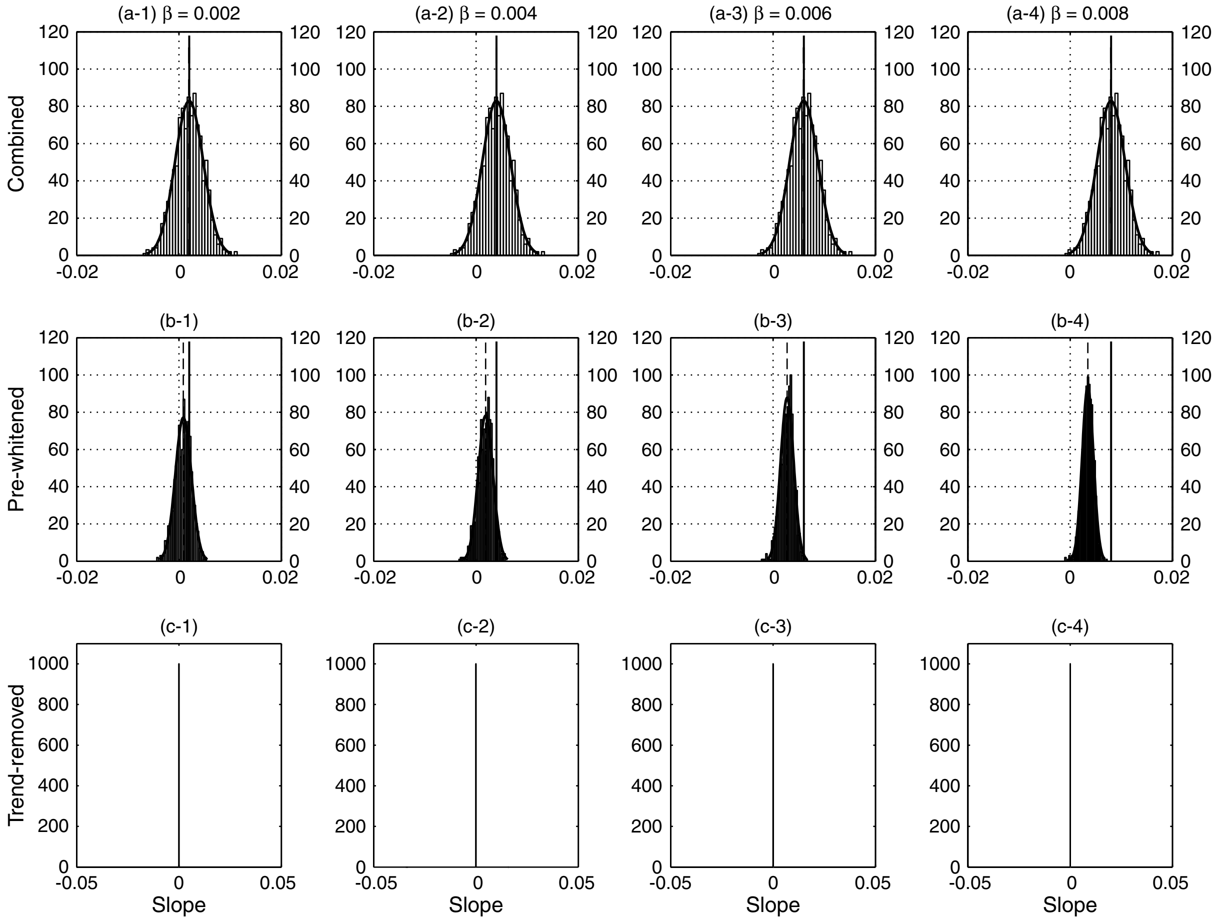

Figure 13. Slope of the trend of the combined series, pre-whitened series, and trend-removed series ($\rho_1 = 0.4$)

The proposed procedure assumes that a hydrological time series can be represented by a linear trend ($T_t = bt$) and an AR(1) component with noise. There are two reasons for only taking these simplest processes into account. First, much of the existing literature investigating the possibility of change has assumed it to be gradual and monotonic. In certain cases (e.g. Gan, 1991, 1998; Gan and Kwong, 1991; Lettenmaier *et al.* 1994; Zhang *et al.*, 2000) there has been some evidence that supports this assumption. Hydrological response to gradual changes could also be gradual. On the other hand, marked shifts in environmental response have also been noted (e.g. Demaree and Nicolis, 1990; Kiely *et al.*, 1998). Given the possibility that certain change may be manifested as a quasi-linear trend, it was felt that further investigation on the abilities and limitations of commonly used statistical procedures to detect such anomalies was warranted. The scope of this study is limited to processes that can be approximated by a linear trend. Second, most of hydrological time series, such as annual streamflows, have relatively weak serial correlation (e.g. Yevjevich, 1963; McMahon and Mein, 1986) and their stochastic behaviour can therefore be approximated by an AR(I) process.

CASE STUDY

The procedures outlined in this study were employed to detect the possible existence of a monotonic trend in serially correlated annual mean and annual minimum daily streamflow data from some river basins in

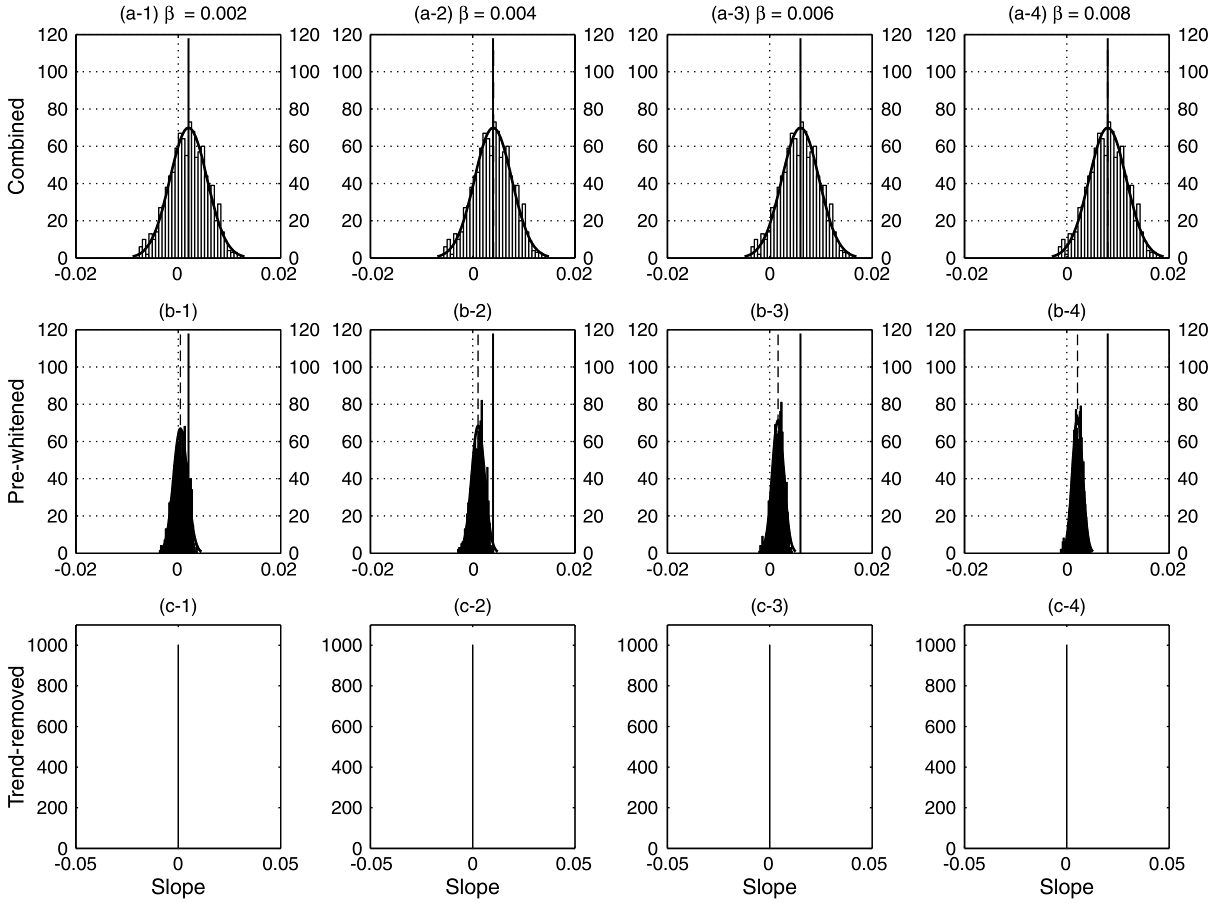

Figure 14. Slope of the trend of the combined series, pre-whitened series, and trend-removed series ($\rho_1 = 0.6$)

Ontario, Canada. These procedures include the MK test on original sample data, the MK test on the sample data that are subjected to von Storch's pre-whitening procedure (MK-PW), the MK test adjusted by the VCA (MK-VCA), and the MK test with the TFPW procedure (MK-TFPW). The daily mean flow data for the basins are recorded in the HYDAT CD-ROM of Environment Canada (1998) and form part of the Canadian Reference Hydrometric Basin Network (RHBN) for climate change studies (Environment Canada, 1999; Pilon and Kuylenstierna, 2000). Watersheds in the RHBN have been selected to meet a number of criteria, such as stable or pristine conditions.

Lag-1 serial correlation coefficients of annual mean and annual minimum daily flows were estimated using Equation (14a). At the significance level of 0.10 for the two-tailed test, the statistical significance of the lag-1 serial correlation coefficients was assessed using Equation (B·1) in Appendix B. As the ability of the TFPW approach is limited to a time series within which a trend can be approximated to be linear, those sites with both a significant serial correlation and a linear trend were selected. The judgement of sample data with a linear trend was made by visual observation of the plot of annual flow versus its occurrence year (these flow diagrams were omitted owing to space limitation). These sites are presented in Table I. The sample data are unitized prior to conducting statistical tests (see Appendix A). This treatment provides a comparable basis for assessing different trends. Columns (2) through (6) of Table I present the lag-1 serial correlation coefficient r_1 estimated using Equation (14a), the upper and lower limits the confidence interval at the significance level

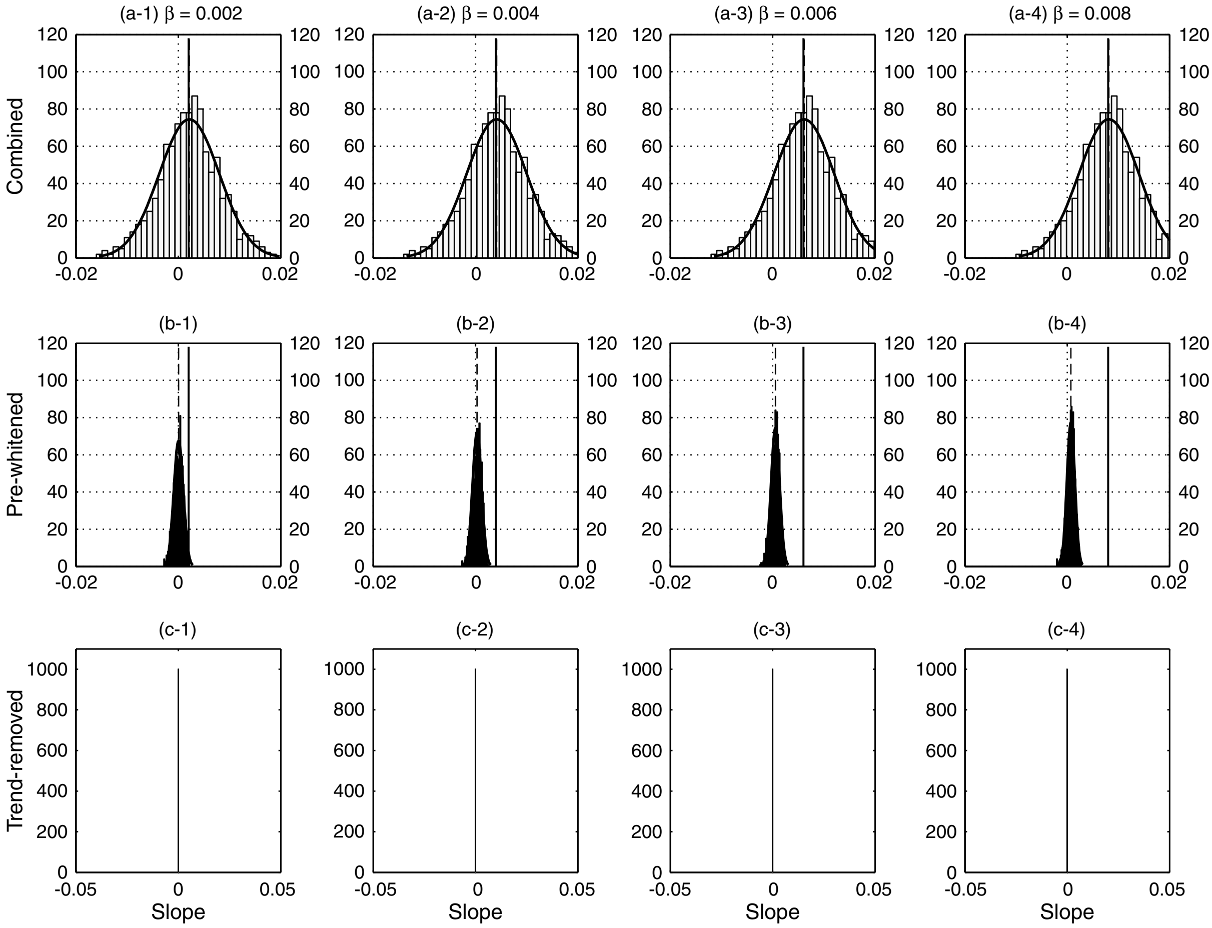

Figure 15. Slope of the trend of the combined series, pre-whitened series, and trend-removed series ($\rho_1 = 0.8$)

of 0.10, by Equation (B·1) in Appendix B, the slope b computed by Equation (17), and the percent increase over 100 years (Pc) by Equation (A·2) of Appendix A, of the unitized annual mean and minimum flows.

The significance of trends in these sites was assessed by the MK test, the MK-PW, the MK-VCA, and the MK-TFPW. The standardized Z values of the MK test and the P values by Equation (6) for the annual mean and annual minimum flows are given in columns (7) and (8) respectively of Table I. The lag-1 serial correlation, the slope, the standardized MK statistic, and P values of the residuals resulting from the pre-whitening by the MK-PW are presented in columns (9) through (13) respectively. From column (9), it can be seen that the von Storch pre-whitening effectively removes the AR(1) process from most sites. By comparing columns (10) and (11) with columns (5) and (6), it is evident that the slopes in the sites after the von Storch pre-whitening are much smaller than those before the von Storch pre-whitening, i.e. removal of positive serial correlation from sample data by pre-whitening also removes a portion of trend. Hence, von Storch pre-whitening will lead to an underestimation of the significance of trends. This effect is reflected in the Z and P values as listed in columns (12) and (13). This result is the same as that observed in the simulation experiments.

For the MK-VCA, the values of the correction factor n/n^*, the modified standardized MK statistic Z^*, and the corresponding P value are presented in columns (14) through (16) respectively in Table I. By comparing the Z^* and P values of the VCA with those of the original MK test listed in columns (7) and (8), it is

Table I. Comparison of the trend assessment results by the MK, MK-PW, MK-VCA, and MK-TFPW

Station identifier	r_1	Confidence limits (0·10)		Slope		MK test											
						Original[a]		von Storch-PW[b]					VCA[c]			TFPW[d]	
		Upper	Lower	b	Pc	Z	P	r_1	Slope		Z	P	n/n^*	Z^*	P	Z	P
									b	Pc							
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
Annual mean daily streamflow																	
02FB007	0·2	0·207	-0·245	0·003 79	38	1·7	0·96	0·02	0·002 28	23	0·94	0·83	0·69	2·07	0·98	1·47	0·93
02GA010	0·2	0·212	-0·253	0·004 62	46	1·8	0·96	-0·01	0·003 04	30	1·04	0·85	1·05	1·73	0·96	1·49	0·93
02HL004	0·3	0·232	-0·282	0·005 65	57	1·8	0·96	-0·03	0·002 17	22	0·62	0·73	1·36	1·53	0·94	1·49	0·93
02KB001	0·2	0·168	-0·193	0·001 56	16	1·5	0·94	0·03	0·001 08	11	0·98	0·84	0·26	2·99	1·00	1·31	0·90
Annual minimum daily streamflow																	
02EA005	0·2	0·168	-0·193	0·001 77	18	0·9	0·81	-0·01	0·001 73	17	0·63	0·74	0·37	1·45	0·93	0·78	0·78
02FB007	0·7	0·207	-0·245	0·025 39	254	6·1	1·00	-0·28	0·007 00	70	1·69	0·95	1·37	5·17	1·00	6·10	1·00
02GA010	0·5	0·212	-0·253	0·013 56	136	4·6	1·00	-0·19	0·004 79	48	1·72	0·96	0·65	5·69	1·00	4·63	1·00
02HL004	0·4	0·232	-0·282	0·016 20	162	2·2	0·99	-0·01	0·007 72	77	1·11	0·87	1·11	2·08	0·98	1·85	0·97

[a] MK test applied to the original sample data.
[b] MK test applied to the sample data subjected to the von Storch pre-whitening procedure.
[c] Variance correction approach.
[d] MK test applied to the sample data subjected to the TFPW procedure.

obvious that the VCA cannot properly correct the influence of the presence of serial correlation on the MK test. Theoretically, for positively correlated series, the correction factor should be greater than one. For the sites 02FB007, 02KB001, 02EA005, and 02GA010, the VCA produces even larger Z values and this leads to worse assessment results than those obtained by the original MK test.

The assessment results by the MK-TFPW are presented in columns (17) through (18) of Table I. By comparing the Z and P values of the blended series by the MK-TFPW with those of the MK test in columns (7) and (8), the MK-PW in columns (12) and (13), and the MK-VCA in columns (15) and (16), it is obvious that the proposed TFPW procedure provides a better assessment of the significance of the trends for serially correlated data than the other approaches.

CONCLUSIONS

This study investigated the interactions between a linear trend and an AR(1) process when both exist in a time series. The Monte Carlo simulation experiments demonstrated that the existence of serial correlation alters the variance of the MK statistic, whereas it does not alter the central tendency or mean and the distribution type of the MK statistic. It was found that positive serial correlation increases the variance of the MK statistic; this increases the probability of detecting a significant trend, whereas in fact none may exist. The existence of a trend influences the magnitude of the estimate of serial correlation.

It was shown that the pre-whitening approach advocated in the previous literature can effectively remove the AR(1) component. However, should a trend exist, pre-whitening a positive AR(1) will also remove a portion of the trend; hence, the slope of the trend after pre-whitening is smaller than the slope before pre-whitening. This will lead to an underestimation of a significant trend. This study has demonstrated that detrending the series prior to pre-whitening provides a more accurate estimate of the true AR(1).

Based on the observations derived from the simulation experiments, an alternative approach, termed the TFPW, is proposed for detecting a significant trend in serially correlated series. This new approach comprises four steps. First, the slope of a trend in the sample data is computed by the TSA. Second, if the slope differs from zero, the identified trend is assumed to be linear and is removed from the sample data. This results in the creation of a residual series that is referred to as the detrended series. Third, the lag-1 serial correlation coefficient of the detrended series is computed, and the AR(1) process is removed from the series. This modified residual series, which results from application of the TFPW procedure, should be an independent series. Finally, the identified trend and the modified residual series are combined, and the MK test is applied to this combined series to assess the significance of a trend.

To further assess the validity of the TFPW procedure in comparison with other trend analysis procedures (MK, MK-PW, and MK-VCA), these procedures were applied to assess the significance of trends in the serially correlated annual mean and annual minimum daily streamflow data from some river basins in Ontario. The results of this analysis proved to be similar to what was established through the Monte Carlo simulation experiments. That is, the MK test, when applied to positively serially correlated series, tends to overestimate the significance of trend. The MK-PW procedure tends to remove a portion of the trend in addition to the AR(1) component when a trend is evident in the original series; this, in turn, results in an underestimation of the significance of a trend in the original series. The MK-VCA cannot effectively correct the influence of serial correlation on the MK test. The TFPW approach developed in this study provides an improved assessment of the significance of trend in such data. The MK test is still advocated for detection of trend for time series that do not have significant serial correlation.

ACKNOWLEDGEMENTS

This study is part of the project 'Use of Statistical Techniques for Assessing the Impact of Climate Change on Streamflow in Canada', sponsored by Environment Canada. The comments made by anonymous reviewers proved useful in improving the quality of the paper.

APPENDIX A

The original sample data X_t is unitized by dividing it by the sample mean $E(X_t)$ as follows:

$$X_t^{\mathrm{u}} = \frac{X_t}{E(X_t)} \quad (A\cdot 1)$$

By this treatment, the mean of each data set is equal to one and the properties of the original sample data remain unchanged. For example, data with record length $n = 100$ years, and a linear slope $b = 0{\cdot}01$, would have a percent increase Pc:

$$\mathrm{Pc}(\%) = \frac{b \times 100}{E(X_t^{\mathrm{u}})} \times 100 = \frac{b \times 10\,000}{1} = 100 \quad (A\cdot 2)$$

APPENDIX B

To judge if observed sample data are serially correlated, the significance of the lag-1 serial correlation at the significance level of $\alpha = 0{\cdot}10$ of the two-tailed test is assessed using the following approximation (Anderson, 1942; Yevjevich, 1972; Salas *et al.*, 1980):

$$\frac{-1 - 1{\cdot}645\sqrt{n-2}}{n-1} \leq r_1 \leq \frac{-1 + 1{\cdot}645\sqrt{n-2}}{n-1} \quad (B\cdot 1)$$

If the lag-1 serial correlation computed by Equation (14a) falls within the confidence interval given by Equation (B.1), the sample data are assumed to be serially independent. Otherwise the sample data are considered to be significantly serially correlated.

REFERENCES

Anderson RL. 1942. Distribution of the serial correlation coefficients. *Annals of Mathematical Statistics* **13**(1): 1–13.

Bayley GV, Hammersley JM. 1946. The effective number of independent observations in an autocorrelated time series. *Journal of the Royal Statistical Society* **8**(1B): 184–197.

Burn DH. 1994. Hydrologic effects of climatic change in West Central Canada. *Journal of Hydrology* **160**: 53–70.

Burn DH, Hag Elnur MA. 2001. *Climate change impact using hydrologic variables*. Report for CCAF project, Environment Canada.

Chiew FHS, McMahon TA. 1993. Detection of trend or change in annual flow of Australian rivers. *International Journal of Climatology* **13**: 643–653.

Demaree GR, Nicolis C. 1990. Onset of Sahelian drought viewed as a fluctuation-induced transition. *Quartely Journal of the Meteorological Society* **116**: 221–238.

Douglas EM, Vogel RM, Kroll CN. 2000. Trends in floods and low flows in the United States: impact of spatial correlation. *Journal of Hydrology* **240**: 90–105.

Environment Canada. 1998. HYDAT CD-ROM Version 98-1.05.8. *Surface water and sediment data*.

Environment Canada. 1999. *Establishment of the Reference Hydrometric Basin Network (RHBN) for Canada*.

Gan TY. 1991. Finding trends in air temperature and precipitation for Canada and north-eastern United States. In *Using Hydrometric Data to Detect and Monitor Climatic Change. Proceedings of NHRI* Workshop No. 8, Gite GW, Harvey KD (eds). National Hydrology Research Institute: Saskatoon, Sask.; 57–78.

Gan TY. 1998. Hydroclimatic trends and possible climatic warming in the Canadian Prairies. *Water Resources Research* **34**(11): 3009–3015.

Gan TY, Kwong YTJ. 1991. Identification of warming trends in northern Alberta and southern northwest territories by the non-parametric Kendall's test. In *Using Hydrometric Data to Detect and Monitor Climatic Change*. Proceedings of NHRI Workshop No. 8, Gite GW, Harvey KD (eds). National Hydrology Research Institute: Saskatoon, Sask.; 43–56.

Hamed KH, Rao AR. 1998. A modified Mann–Kendall trend test for autocorrelated data. *Journal of Hydrology* **204**: 182–196.

Hirsch RM, Slack JR. 1984. Non-parametric trend test for seasonal data with serial dependence. *Water Resources Research* **20**(6): 727–732.

Hirsch RM, Slack JR, Smith RA. 1982. Techniques of trend analysis for monthly water quality data. *Water Resources Research* **18**(1): 107–121.

Kendall MG. 1975. *Rank Correlation Methods*. Griffin: London.

Kendall SM, Ord JK. 1990. *Time Series*, 3rd edition. Edward Arnold of Hodder and Stoughton Limited: UK.

Kiely G, Albertson JD, Parlange MB. 1998. Recent trends in diurnal variation of precipitation in Valentia on west coast of Ireland. *Journal of Hydrology* **207**: 270–279.

Kulkarni A, von Storch H. 1995. Monte Carlo experiments on the effect of serial correlation on the Mann–Kendall test of trend. *Meteorologische Zeitschrift* **4**(2): 82–85.

Mann HB. 1945. Nonparametric tests against trend. *Econometrica* **13**: 245–259.

McMahon TA, Mein RG. 1986. *River and Reservoir Yield*. Water Resources Publications: Littleton, CO, USA.

Lettenmaier DP. 1976. Detection of trends in water quality data from records with dependent observations. *Water Resources Research* **12**(5): 1037–1046.

Lettenmaier DP, Wood EF, Wallis JR. 1994. Hydro-climatological trends in the continental United States, 1948–88. *Journal of Climate* **7**: 586–607.

Lins HF, Slack JR. 1999. Streamflow trends in the United States. *Geophysical Research Letters* **26**(2): 227–230.

Pilon PJ, Kuylenstierna JK. 2000. Pristine river basins and relevant hydrological indices: essential ingredients for climate-change studies. *WMO Bulletin* **49**(3): 248–255.

Salas JD, Delleur JW, Yevjevich V, Lane WL. 1980. *Applied Modelling of Hydrologic Time Series*. Water Resources Publications: Littleton, CO, USA.

Salas JD. 1993. Analysis and modeling of hydrologic time series. In *Handbook of Hydrology*, Maidment DR (ed.). McGraw-Hill: New York; Chapter 19.

Sen PK. 1968. Estimates of the regression coefficient based on Kendall's tau. *Journal of the American Statistical Association* **63**: 1379–1389.

Theil H. 1950a. A rank-invariant method of linear and polynomial regression analysis, I. *Nederlands Akad. Wetensch. Proc.* **53**: 386–392.

Theil H. 1950b. A rank-invariant method of linear and polynomial regression analysis, II. *Nederlands Akad. Wetensch. Proc.* **53**: 521–525.

Theil H. 1950c. A rank-invariant method of linear and polynomial regression analysis, III. *Nederlands Akad. Wetensch. Proc.* **53**: 1397–1412.

Thiebaux HJ, Zwiers FW. 1984. The interpretation and estimation of effective sample size. *Journal of Climate and Applied Meteorology* **23**: 800–811.

Von Storch VH. 1995. Misuses of statistical analysis in climate research. In *Analysis of Climate Variability: Applications of Statistical Techniques*, von Storch H, Navarra A (eds). Springer-Verlag: Berlin, 11–26.

Wei WWS. 1990. *Time Series Analysis: Univariate and Multivariate Methods*. Addison-Wesley Publishing Company, Inc.

Yevjevich V. 1963. *Fluctuations of wet and try years, part I, research data assembly and mathematical models*. Hydrology paper 1, Colorado State University, Fort Collins, USA.

Yevjevich V. 1972. *Stochastic Processes in Hydrology*. Water Resources Publications: Fort Collins, CO, USA.

Yulianti JS, Burn DH. 1998. Investigating links between climatic warming and low streamflow in the Prairies Region of Canada. *Canadian Water Resources Journal* **23**(1): 45–60.

Zhang X, Vincent LA, Hogg WD, Niitsoo A. 2000. Temperature and precipitation trends in Canada during the 20th century. *Atmosphere–Ocean* **38**(3): 395–429.

Zhang X, Harvey KD, Hogg WD, Yuzyk TR. 2001. Trends in Canadian streamflow. *Water Resources Research* **37**(4): 987–998.

